Probabilistic seismic response analysis of a 3-D reinforced concrete building

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\begin{abstract}
This paper presents the probabilistic seismic demand analysis with respect to seismic input uncertainty only of a 3-D reinforced concrete building model subjected to three-component earthquake ground motion excitation. Response history analyses are carried out on a nonlinear frame model. Probability distributions are assumed for the ground motion Intensity Measure (IM) taken as the linear 5% damped elastic spectral acceleration at the fundamental period of the structure. Part of the framework of the so-called Performance Based Earthquake Engineering (PBEE) methodology developed by the Pacific Earthquake Engineering Research (PEER) Center is used in this study. This paper has two main objectives. The first objective is to perform a probabilistic demand analysis of an existing building conditional on the ground motion IM. The second objective is to use the results obtained from this existing testbed, with real-world complexities, to demonstrate the deficiency of the PEER PBEE methodology when using a scalar ground motion IM for 3-D structural models. This last objective shows the need for improving the definition of the seismic IM in the PBEE methodology for the general case of 3-D structures subjected to multi-directional input ground motions. To this effect, an ensemble of natural ground motion records is used to represent the inherent randomness in ground motion time-histories (i.e., record-to-record variability). The statistical correlation of different Engineering Demand Parameters (EDPs) with a set of IMs, taken as the 5% damped spectral accelerations at different periods for two horizontal ground motion components, is investigated in order to assess the dispersion in the EDPs due to different ground motion records. Some statistical correlation coefficients are found to be high, indicating that the dispersion of the EDPs is heavily influenced by the spectral content at periods different from the fundamental period. This result points to the need for using vector-valued ground motion Intensity Measures in the PBEE methodology currently proposed by PEER.
\end{abstract}

\section{Introduction}

In the last decade, the Pacific Earthquake Engineering Research (PEER) center has focused on the development of methodologies and tools for Performance-Based Earthquake Engineering of building and bridge structures [1–5]. PBEE stands on the premise that performance can be predicted and evaluated with quantifiable confidence in order to make intelligent and informed decisions based on life-cycle considerations. To achieve this objective, PEER has focused on methods of performance assessment, with due consideration to the effects of all pertinent uncertainties that enter the performance prediction process, from earthquake occurrence modeling to the assessment of earthquake consequences such as casualties, dollar losses and downtime [6,7]. The methodology has been applied to existing facilities (e.g., buildings, bridges, network of highway bridges, and campuses of buildings) within the PEER testbed program. Since the ground shaking intensity and the mechanical characteristics of a building are both highly uncertain, it is not possible to calculate deterministic values of the seismic structural demands. Instead, it is necessary to predict a statistical distribution of the demand parameters, considering the variability in the input (ground shaking intensity and ground motion characteristics), in the structural model itself and in its various parameters. Furthermore, traditional linear structural analysis provides for the least accurate prediction of the seismic demand. Rigorous characterization of the building behavior should be obtained using detailed nonlinear models in order to perform a reliable probabilistic seismic demand assessment.

The PBEE methodology developed by PEER integrates the following four steps in a unified probabilistic framework: (1) seismic hazard analysis, (2) probabilistic demand analysis, (3) fragility analysis, and (4) loss analysis. This framework uses as interface variables the ground motion Intensity Measure (IM) and the Damage Measure (DM). Typically, the IM is the 5% damped elastic spectral acceleration at the fundamental period of the structure $T_1$, and DMs are discrete
limit-states, such as yielding of longitudinal reinforcement, spalling of concrete, buckling of longitudinal reinforcement, etc. Ideally, the IM should satisfy the attributes of efficiency and sufficiency [8]. This paper deals only with two of the analytical steps of the PEER PBEE methodology, namely (1) probabilistic seismic hazard analysis (PSHA) and (2) probabilistic seismic demand analysis (PSDA).

PBEE, the result of PSHA is the probabilistic characterization of a scalar or vector-valued ground motion intensity Measure (IM). This characterization of the IM is used to quantify the likelihood of a ground motion (i.e., annual probability of exceedance) at a site. The IM is also used as a seismological variable to which the EDPs of the structure are correlated, with these EDPs obtained through nonlinear time history analyses of a structural model subjected to carefully selected ground motions records. Two properties were defined to be of crucial importance in the selection of the IM [8]: (1) efficiency and (2) sufficiency. An efficient IM is defined as one that results in a relatively low variability of the EDP conditional on this IM. A sufficient IM is defined as one that renders the EDP conditionally statistically independent of other ground motion characteristics given this IM. Historically, the peak ground acceleration (PGA) was initially used as the IM in PSHA [9]. In this paper, however, the selection of \( S_0(T_1) \) as the primary IM follows the work of Cornell and co-workers who showed that \( S_0(T_1) \) was more “efficient” and more “sufficient” than PGA [8,10]. Nonetheless, recent studies based on 2-D structural models, have demonstrated that \( S_0(T_1) \) may not be particularly “efficient” nor “sufficient” for some structures and ground motion types (e.g., tall, long-period buildings [11], near-source ground motions [12]).

Among other researchers, Lee and Mosalam [13] used part of the PEER methodology to perform a probabilistic seismic demand analysis of a reinforced concrete building modeled in 2-D, and to investigate the probabilistic structural demand sensitivity to uncertainties in the structural model parameters and in the ground motions. However, limited information is available on the seismic response of 3-D building structures under multi-component earthquake ground motion input. In case of 3-D structural behavior, it is difficult, if not impossible, to use a scalar IM that satisfies both requirements of efficiency and sufficiency.

An important issue when dealing with nonlinear time-history analyses is the use of a scalar versus vector-valued IM, and the subsequent adoption of a proper scaling criterion of the input ground motions. Within PEER, a strategic decision was made to use historical ground motion records scaled within reasonable limits as opposed to synthetic or simulated ground motions. In the PEER PBEE methodology, the spectral ordinate \( S_0(T_1) \) of the ground acceleration component in the direction of the fundamental mode with period \( T_1 \) on the uniform hazard spectrum has been adopted as an optimal Intensity Measure for a suite of records. The structural response is strongly correlated to this IM for some response parameters and for some classes of structures [14]. In PBEE, the input motions are typically scaled to the value of \( S_0(T_1) \) corresponding to the specified hazard level. This scaling criterion has also been extended for use in the analysis of 3-D structures, with the same scaling applied to all three seismic input components. There is concern regarding the adequacy of such an assumption for 3-D analysis of structures. Thus, even though there has been significant research on ground motion selection and scaling [15–18], the application of PBEE based on 3-D structural models and analyses for multi-component ground motion excitations, as described in this paper, deserves more attention.

The first objective of this paper is to show the need for improving the definition of the seismic IM in the PBEE methodology for the general case of 3-D structures subjected to multi-directional input ground motion. The second objective of this paper is to use an existing tested structure to demonstrate the deficiency of PBEE when using a scalar ground motion IM in conjunction with a 3-D model of the structure. A nonlinear 3-D frame model of an existing reinforced concrete (R/C) structure built in 1984 in Bonferr, Italy, and damaged in the 2002 Molise earthquake [19] is used as a case study for illustrating the implications of using a scalar IM in the PEER PBEE methodology, making use of the structural analysis software framework OpenSees [20,21]. Probability distributions are assumed for the ground motion IM \( S_0(T_1) \). An ensemble of natural ground motion records selected within the PEER testbed program for the campus of the University of California at Berkeley is used to represent the inherent randomness in ground motion time-histories (i.e., record-to-record variability) in the Monte Carlo simulations. A probabilistic study is performed of the structural response parameters, referred herein as Engineering Demand Parameters (EDPs).

2. Analytical model of the Bonferr building

The building used in this study is an existing four-story reinforced concrete structure built in the early 1980s in Bonferr, Italy, and shown in Fig. 1. The building is an example of typical building practices used in Italy for residential structures before modern seismic codes were introduced, and has a structural layout and design details that can be found in older buildings in several seismic regions throughout the world. The ground level columns and masonry infills suffered severe damage during the 2002 Molise earthquake [19], while the upper stories remained substantially undamaged. The structural system comprises three frames in the longitudinal direction (X-direction). Shallow and wide beams are used in the central longitudinal frame. One-way slabs connect the longitudinal frames (Fig. 1(b)). In the short direction (Y-direction), there are four frames. The building is plan symmetric in the Y-direction, while the staircase and the ground level infills break the symmetry in the X-direction. Further information on dimensions of the structural elements and their reinforcement detailing can be found in [23].

A 3-D nonlinear model of the building structure was created in OpenSees and is shown in Fig. 2(a). The concrete frame behavior is affected by other structural components, such as the infills, floor diaphragms, beam-column joints, staircase, etc. [22,23]. The effects of such components on the structural response, which could potentially be very important, are not studied in this paper, as the main objective here is advancing current procedures for probabilistic seismic response analysis of 3-D building structures. Furthermore, a simplified building configuration reduces the significant computational effort necessary to perform large ensembles of time-history analyses. For these reasons, only the bare frame (beams and columns) is considered in the model used herein, with fixed supports at the building base. Floor diaphragms rigid in their plane are used to model the floor slab. Rheological effects, such as volumetric changes caused by creep, shrinkage and temperature, have also been disregarded in this model. Since it was found early in this study that nonlinear geometry has a negligible effect on the EDPs considered here (defined as peak response quantities), geometric nonlinearity was neglected in the analyses. The study was carried out using a high-performance computational environment that relies on parallel supercomputers for parametric sensitivity analyses. This environment was developed in a related study [24] and makes use of OpenSees Version 1.7.3 [25] which is suited for parallel computations.

In the building model, beams and columns are modeled using force-based nonlinear frame elements [26]. Each element has five Gauss–lobatto integration points (i.e., monitored cross sections) along the element length. This number was selected in order to match the plastic hinge length of the columns with the tributary length of the first integration point in order to circumvent localization issues and loss of objectivity in the simulated response [27]. At each integration point, the section is discretized using a fiber model. The uniaxial Kent and Park [28] concrete model is used for both the unconfined and confined concrete, with no tensile strength. The steel rebars are modeled using a bilinear inelastic law. The cross-sectional response, based on the fiber discretization, is capable of capturing
the stiffness degradation and strength deterioration due to concrete cracking, concrete crushing, and steel yielding, based on the uni-axial material constitutive laws adopted (see sketches in Fig. 2(c)). The fiber cross-section automatically accounts for the interaction between axial force and biaxial bending. Cross-section and element shear and torsional behaviors are assumed linear elastic, uncoupled, and aggregated to the nonlinear inelastic fiber cross-section behavior. Thus, the beam-column elements do not account for stiffness degradation and strength deterioration in the shear and torsional behavior of the beams and columns.

All material and structural properties are taken at their best deterministic estimates. The concrete cube strength $f_{cm,cube}$ is set to the mean test value of 25.8 MPa. Similarly, the steel strength $f_y$ is set to 451 MPa. The mean values assumed for the concrete and steel Young’s modulus are 28.9 and 210 GPa, respectively. Rayleigh damping was assumed based on the initial stiffness matrix of the structure (after application of the gravity loads) with a 3.0% damping ratio in the initial modes 1 and 6 with frequencies $f_1 = 0.93$ Hz and $f_6 = 4.0$ Hz. Since the hysteretic energy dissipation is already explicitly accounted for at the material level, the damping ratio used is lower than the 5.0%
value commonly suggested for linear analyses. The model is symmetric in the Y-direction and has a small asymmetry in the X-direction, as shown in Fig. 1. The floor masses used in the dynamic analyses include all dead loads and 30% of the 2 kN/m² live load according to Eurocode 8 [29]. The floor masses are uniformly distributed in plan and along the building height, with a typical floor seismic weight of approximately 1700 kN. It is worth noting that the masses of the infills and of the stairs were also accounted for and included in the uniformly distributed floor masses. The floor mass density at the highest level is 0.87 tons/m². The distributed floor mass was discretized based on tributary areas into lumped masses (of equal values in the X, Y, and Z directions) assigned to each beam-column joint. The rotatory inertia of the floor diaphragms is automatically accounted for by the diaphragm constraint applied to each floor. The dynamic time-history analyses are performed after the model is loaded with the gravity loads during an initial nonlinear static analysis. The constant average acceleration Newmark method was used to integrate the equations of motion. The Newton–Raphson algorithm was used to find convergence of the dynamic equilibrium at each time step. All time-history analyses performed in this study converged.

3. PEER methodology and ground motion Intensity Measure

As mentioned above, the PEER PBEE methodology is a 4-stage seismic assessment procedure for facilities located in a zone with a given seismicity [30]. The present study involves the first two stages only: (1) probabilistic seismic hazard analysis, and (2) probabilistic structural analysis.

The probabilistic seismic hazard analysis yields the seismic hazard curve, which provides the mean annual rate (MAR) of the Intensity Measure $S_a(T_1)$ exceeding any specified level at the site of interest. An ensemble of actual earthquake ground motion records are selected and scaled so that their spectral ordinates at the initial fundamental period $T_1$ of the building match the Intensity Measure at three hazard levels corresponding to a probability of exceedance (PE) of 50% in 50 years, 10% in 50 years, and 2% in 50 years, respectively.

The probabilistic structural analysis stage consists of performing an ensemble of nonlinear time-history analyses of the finite element (FE) model of the structure at each of the three hazard levels considered. The uncertainty in the structural response is measured by monitoring the scatter of a number of structural response parameters called EDPs at each of the three hazard levels.

The present study focuses only on the second stage of the PEER PBEE methodology, mainly on the probabilistic seismic demand analysis (structural analysis). The probabilistic approach follows the propagation of the seismic input uncertainty to a number of EDPs through the analysis of a nonlinear model of the building structure. The results are given in terms of the likelihood of the various EDPs exceeding specified demand levels. In this paper, we consider only the uncertainty in the seismic input and focus on probabilistic demand analyses conditional on IM. The seismic input uncertainty consists of the uncertainty in the IM (or in the seismic excitation ratio) and the randomness in the ground motion time-history (record-to-record variability). At each hazard level, the record-to-record variability is taken into account by performing Monte Carlo simulations with an ensemble of 20 scaled tri-directional records.

The ground motions used in this study are those selected within the PEER testbed program (http://peer.berkeley.edu/research/peertestsbeds/index.html) for the site at the University of California, Berkeley [13,31], and are reported in Table 1. Each record consists of two horizontal and one vertical component. Thus, these records were not selected for the specific site where the Bonefro building is actually located, as the Bonefro building is used here as a mere testbed structure outside of its seismic environment. The site for which the ground motions are selected is located in proximity of the Hayward fault. The 20 ground motions were selected by Somerville [31] for the PEER testbed program. Even though these records are from different regions of the world, their seismological parameters are similar to those of the Hayward fault. Furthermore, the ground motion selection is consistent with the magnitude–distance (M–R) deaggregation of the seismic hazard at the site based on IM = $S_a(T_1)$ and with the type of faulting mechanism and local site conditions. Most applications of the PEER PBEE methodology have used a scaling criterion of the input seismic records based on the single scalar IM = $S_a(T_1)$.

The initial first six modal periods of the building in the bare frame configuration are $T_1$ = 1.07 s, $T_2$ = 0.87 s, $T_3$ = 0.77 s, $T_4$ = 0.33 s, $T_5$ = 0.28 s, $T_6$ = 0.25 s. With the discretization of the masses adopted in the present model, the most significant vertical mode of vibration is mode 17, with a period of $T_{17}$ = 0.05 s and a modal participating mass ratio of 30%. These periods were computed based on the stiffness of the structure after applying the gravity loads and before starting the nonlinear time-history analyses. Periods $T_1$, $T_2$, and $T_3$ correspond to the first sway mode in the transversal/short Y-direction, the first sway mode in the longitudinal/long X-direction, and the first torsional mode, respectively. Periods $T_4$, $T_5$, and $T_6$ correspond to the second sway mode in the Y-direction, the second sway mode in the X-direction, and the second torsional mode, respectively. The ground motions are applied so that the component referred to as “longitudinal” by Somerville [31] acts parallel to the building Y-direction. The records were scaled so as to yield the same ordinate $S_a(T_1)$ in the Y component and the same scaling factor was used for all three ground motion components of a given record. The scaling factors for the ground motions used ranged from 0.10 to 1.33 (with a mean of 0.39 and standard deviation of 0.33) for the 50% in 50 years hazard level and from 0.26 to 3.75 (with a mean of 1.12 and standard deviation of 0.93) for the 10% in 50 years hazard level.

The probabilistic seismic hazard analysis for the Berkeley site provides uniform hazard spectra at the three hazard levels (50%, 10%, and 2% annual probability of exceedance (PE) in 50 years, corresponding to 72, 475 and 2475 years return period, respectively, assuming the Poisson model for random earthquake occurrences). These uniform hazard spectra are provided for a discrete number of periods, as shown in Fig. 3, and the values of $S_a(T_1)$ at intermediate period values are obtained through log-linear interpolation. The ordinates of the three uniform hazard spectra at period $T_1$ = 1.07 s in Fig. 3, together with the corresponding values of the MAR of exceedance, are plotted as the three white-filled points in Fig. 4, which shows the MAR of exceedance (or the annual PE) as a function of the IM $S_a(T_1)$.

In Fig. 4, these three white-filled points are then converted into the black points that represent the probability of exceedance in 50 years assuming the Poisson random occurrence model. A lognormal complementary cumulative distribution function (CCDF) represented by the thick solid line is least-square fitted to the three black points. The fitted lognormal CCDF, which provides the probability of exceedance in 50 years versus $S_a(T_1, \xi = 5\%)$, was transformed again assuming the Poisson random occurrence model to the annual probability of exceedance (dashed line) and the MAR of exceedance (thin solid line) versus $S_a(T_1, \xi = 5\%)$. The annual PE versus IM = $S_a(T_1, \xi = 5\%)$ are referred to in the literature as the seismic hazard curve for the site. It is worth noting that the seismic hazard curves in terms of annual probability of exceedance and MAR of exceedance coincide except at low hazard levels (high probability of exceedance). Using the site seismic hazard curve, the ground motion IM can be set to any hazard level and the selected ground motions scaled accordingly.

Assuming that earthquake ground motions with IM exceeding the threshold $\mathrm{IM} = \lambda(\mathrm{im})$ follow a Poisson random occurrence model in time, the probability of one or more ground motions with $\mathrm{IM} > \lambda(\mathrm{im})$ during an exposure time $\tau$ is given by [32]

$$P = 1 - e^{-\lambda(\mathrm{im})\cdot\tau} \quad (1)$$

where $\lambda(\mathrm{im})$ denotes the MAR of occurrence of ground motions with $\mathrm{IM} > \lambda(\mathrm{im})$. The thin solid line in Fig. 4 represents $\lambda(\mathrm{im})$ with im defined.
Table 1
Ground motion recordings for the site of the UC Berkeley Science Building [31].

<table>
<thead>
<tr>
<th>Record</th>
<th>Event</th>
<th>Station</th>
<th>Year</th>
<th>Δt</th>
<th>Npoints</th>
</tr>
</thead>
<tbody>
<tr>
<td>EZ_eri</td>
<td>Erzincan</td>
<td>Erzincan</td>
<td>1992</td>
<td>0.005</td>
<td>4156</td>
</tr>
<tr>
<td>KB_kobj</td>
<td>Kobe</td>
<td>Kobe JMA</td>
<td>1995</td>
<td>0.02</td>
<td>2499</td>
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<tr>
<td>LP_cor</td>
<td>Loma Prieta</td>
<td>Corralitos</td>
<td>1989</td>
<td>0.005</td>
<td>7989</td>
</tr>
<tr>
<td>LP_gav</td>
<td>Loma Prieta</td>
<td>Gavilan College</td>
<td>1989</td>
<td>0.005</td>
<td>7991</td>
</tr>
<tr>
<td>LP_gilb</td>
<td>Loma Prieta</td>
<td>Gilroy historic</td>
<td>1989</td>
<td>0.005</td>
<td>7991</td>
</tr>
<tr>
<td>LP_lexx</td>
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<td>Lexington Dam</td>
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<td>0.02</td>
<td>2000</td>
</tr>
<tr>
<td>LP_lgc</td>
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<td>Los Gatos Res. Center</td>
<td>1989</td>
<td>0.005</td>
<td>4993</td>
</tr>
<tr>
<td>LPlrtg</td>
<td>Loma Prieta</td>
<td>Saratoga Aloha Ave</td>
<td>1989</td>
<td>0.005</td>
<td>7991</td>
</tr>
<tr>
<td>TG_tr007</td>
<td>Tottori</td>
<td>Kofu</td>
<td>2000</td>
<td>0.01</td>
<td>4000</td>
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<tr>
<td>TG_tr012</td>
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<td>Hino</td>
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<td>10000</td>
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<td>CL_chyd</td>
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<td>Coyote Lake Dam</td>
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<td>5764</td>
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<tr>
<td>CL_gilb</td>
<td>Coyote Lake</td>
<td>Gilroy</td>
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<td>0.005</td>
<td>5419</td>
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<tr>
<td>LV_fgnr</td>
<td>Livermore</td>
<td>Fagundes Ranch</td>
<td>1980</td>
<td></td>
<td>4000</td>
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<tr>
<td>LV_mgnp</td>
<td>Livermore</td>
<td>Morgan Territory Park</td>
<td>1980</td>
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<td>4800</td>
</tr>
<tr>
<td>MH_1add</td>
<td>Morgan Hill</td>
<td>Coyote Lake Dam</td>
<td>1984</td>
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<td>MH_chyd</td>
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</table>

Fig. 3. Uniform hazard spectra for the UC Berkeley Science Building site. Thin line: linear interpolation between discrete values provided by USGS; thick line: log-linear interpolation between discrete values.

Fig. 4. Lognormal approximation of seismic hazard exceeded in 50 years and corresponding seismic hazard curves for annual probability of exceedance and mean annual rate of exceedance.

as $S_\lambda(T_1, \xi = 5\%)$. The annual probability of exceedance (dashed line in Fig. 4) and the probability of exceedance in 50 years (thick solid line in Fig. 4) versus $IM = S_{g.m.}(T_1)$ are related to $\lambda(\xi)$ through Eq. (1) by setting $t = 1$ and $T = 50$, respectively.

It should be noted that the seismic hazard curve for the fundamental period of the building $T_1$ was obtained for the geometric mean of the spectral accelerations in two horizontal components, $S_{g.m.}(T_1)$, at three hazard levels ($5\%$, $10\%$, and $2\%$ in 50 years) that were used in the determination of the seismic hazard curve. Then, the seismic hazard curve for $S_{g.m.}(T_1)$ was used to characterize the spectral acceleration in the Y-direction of the building at $T_1$, $S_{g.v.}(T_1)$, and to scale the ensemble of three-component ground motion records at two hazard levels, $5\%$ and $10\%$ in 50 years. The scaling factors that were applied to the records in the Y-direction of the building were also applied to the other two components of the record (horizontal component in the X-direction and the vertical component). Therefore, the geometric mean of the spectral accelerations of the two horizontal components of the ensemble of ground motion records is not scaled exactly to the seismic hazard curve $S_{g.m.}(T_1)$. Thus, there is some inconsistency between the Intensity Measure used to define the seismic hazard and the one used in scaling the ground motions, which would need to be prevented or accounted for if a full PBEE analysis (or at least an unconditional probabilistic seismic demand analysis) was performed. Baker and Cornell [15] discuss similar inconsistencies and how they can be prevented or accounted for in a full PBEE analysis. However, for the scope of this paper, this inconsistency does not play a role in the main findings discussed later, which highlight the variation of the correlation coefficients between key response parameters and spectral accelerations in two orthogonal directions of the building structure and at periods other than $T_1$.

4. Propagation of uncertainty in ground motion records to 3-D model EDPs conditional on IM = $S_{g.v.}(T_1)$

In order to properly evaluate the probabilistic characteristics of the EDPs of the 3-D building model, the dynamic analysis should be based on rational assumptions regarding scaling of the three ground motion components defining the seismic input. This section addresses this particular issue which is still in need of a rational solution in the PEER PBEE methodology. Only two hazard levels, mainly corresponding to $5\%$ and $10\%$ PE in 50 years, will be considered. At the lower hazard level the structure is likely to remain mostly elastic, while at the higher hazard level the structure is very likely to undergo significant inelastic deformations and damage.
4.1. Statistics of elastic spectra conditional on IM = $S_{XY}(T_1)$

In the probabilistic structural analysis step of the PBEE methodology, the uncertainties represented by the basic random variables or input parameters (characterizing in general the seismic input loading and the structure) are propagated through the nonlinear FE analysis to the uncertainties characterizing the EDPs. The mapping between input parameters and EDPs is represented by the nonlinear time-history analysis procedure.

The pseudo-acceleration response spectra $S_{XY}(T)$ and displacement response spectra $S_{XY}(T)$ shown in Fig. 5(a) and (b), respectively, show clearly that the ordinates $S_{XY}(T_1)$ and $S_{XY}(T_1)$ are the same (corresponding to a hazard level defined by a PE of 50% in 50 years) for all 20 ground motion components. However, there is a wide dispersion of $S_{XY}(T)$ and $S_{XY}(T)$ at periods $T$ away from the initial fundamental period $T_1$. A large scatter is observed for the pseudo-accelerations and the spectral displacements at the higher mode periods ($T_2$, $T_3$, $T_4$) and for the spectral displacements at periods higher than $T_1$ such as the fundamental period of the building after it has been damaged and has undergone inelastic deformations. In other words, the ground motion scaling criterion used in this study introduces different seismic hazard levels at different periods and in different directions of the seismic input. The circle symbols along the vertical line at $T = T_1 = 0.33$ s in Fig. 5(a) represent the statistical distribution of a different IM = $S_{XY}(T_1) = 0.33$ s obtained from the 20 ground motion components scaled at the PE of 50% in 50 years for $S_{XY}(T_1)$. $T_2$ is the second mode period in the Y-direction of the building. As evidenced in the figure, the circle symbols depart significantly from the corresponding ordinate of the uniform hazard spectrum for 50% PE in 50 years. Therefore, if the scaling criterion based on $S_{XY}(T_1)$ is used, while the main IM = $S_{XY}(T_1)$ has a PE of 50% in 50 years, the spectral accelerations of the scaled ground motion components at periods of interest for the building analysis different from $T_1$ may have a wide range of hazard levels, as measured by the PE in 50 years. The scatter of the spectral ordinates of the scaled ground motion components at periods away from $T_1$ will significantly influence the scatter of the EDPs in the probabilistic response analysis conditional on IM = $S_{XY}(T_1)$, as will be shown later.

Fig. 6(a) and (b) shows the pseudo-acceleration spectra $S_{XY}(T)$ and displacement spectra $S_{XY}(T)$ of the scaled ground motion components in the X-direction of the building. Similarly, Fig. 7(a) and (b) shows the pseudo-acceleration spectra $S_{XY}(T)$ and displacement spectra $S_{XY}(T)$ in the Z (vertical) direction. Since the same scaling factor is used for all three components of each tri-directional record, the spectra of the X and Z ground motion components do not intersect at the point on the uniform hazard spectrum at $T = T_1$. Again, it is observed from Figs. 6 and 7 that the ground motion scaling criterion, based on $S_{XY}(T_1$, $\xi = 5\%$), of the Y-direction ground motion components produces a small scatter in the hazard level of the spectral ordinates at other periods and of other ground motion components (i.e., X and Z-direction components).

4.2. Deterministic local and global response

To illustrate the deterministic response of the building to a single seismic input, this section describes the results of two time-history analyses at two hazard levels for both global and local EDPs. Global EDPs relate to the response at the overall structural level, e.g., displacements and accelerations at floor levels. Local EDPs relate to the response at the level of the structural components (beams or columns), e.g., interstory drift ratios, or sub-components (column or beam section), e.g., section curvature. The following selected results correspond to an EDP specific “median ground motion”. The latter denotes the ground motion record that produces the median EDP among the EDP values produced by the ensemble of 20 ground motion records. For this purpose, the ground motion record TO_trh02 (see Table 1) was scaled to $S_{XY}(T_1 = 1.07$ s) with a 50% and 10% probability of exceedance in 50 years, respectively. This ground motion record corresponds to the median ground motion at the 50% in 50 years hazard level for the local EDP defined as the Max curvature in the Y-direction at the base section of the column labeled 2008 in Fig. 2(a).

A second-story column was selected because under static push-over analysis in the Y-direction, the largest interstory drift occurs in the second story. Furthermore, the two external frames are the stiffest and strongest frames in the Y-direction and therefore, under the assumption of rigid floor diaphragms, attract the highest lateral forces. Finally, the middle column of the external frame was selected since it is the stiffest and strongest column of the frame and therefore the one subjected to the largest flexural and shear forces.

The global seismic response of the building is measured by the floor horizontal displacements relative to the ground at the floor centers of mass in the X and Y-directions and by the corresponding interstory drifts. These response quantities, as well as the maximum floor absolute accelerations at the floor centers of mass in the X and Y-directions, are used as global EDPs in this study.

The deterministic local and global EDPs defined above were recorded for the building subjected to ground motion record TO_trh02 and are shown in Fig. 8 for the 50% PE in 50 years hazard level and in Fig. 9 for the 10% PE in 50 years hazard level. It is observed that the response of the building remains quasi-linear (near incipient yield) at the 50% PE in 50 years hazard level, and is significantly nonlinear at the 10% PE in 50 years hazard level, as shown by the section moment-curvature response plots. Peak interstory drifts of 1.6% and 4.3% are reached at the 50% and 10% PE in 50 years hazard levels, respectively. It is worth noting that at 4.3% of interstory drift, which is rather large for a nonductile concrete frame structure, the main source of deterioration in the nonlinear FE model is related to the post-crushing behavior of concrete. The irregular shape of the hysteretic loops in the section moment-curvature responses reflects the interaction between bi-axial flexure and variable axial force.

4.3. Response ensemble statistics conditional on $S_{XY}(T_1)$ at 50% PE and 10% PE in 50 years hazard levels

The time-history analyses were repeated for each record of the ensemble of the earthquake ground motion records defined earlier to obtain ensemble statistics of the building response at the 50% and 10% PE in 50 years hazard levels. The maximum building response in the X and Y-directions, in terms of peak floor displacements (PFD), peak interstory drift ratios (PIDRs) and peak floor absolute accelerations (PFAs), is shown in Fig. 10 for the 50% PE in 50 years hazard level, and in Fig. 11 for the 10% PE in 50 years hazard level. Each sub-figure in Figs. 10 and 11 displays 20 peak response profiles (in thin solid lines) corresponding to the 20 tri-directional ground motion records considered in this study and the ensemble average of the peak response profiles or mean peak response profile (in thick solid line). It should be noted that, since the columns have relatively low axial load [23], their bending capacity and stiffness decrease from the first to the second story; thus, even though the shear demand decreases along the height of the building, the interstory drift demand is higher at the second story. The scatter of the peak response parameters is expressed graphically in Figs. 10 and 11 and quantified in the form of coefficient-of-variations (c.o.v.) in Tables 2 and 3 for the 50% and 10% PE in 50 years hazard levels, respectively.

At the 50% PE in 50 years hazard level, the columns do not undergo any significant inelastic deformations, and the mean peak interstory drift ratios remain below or near 1% in both the X and Y-directions. The results given in Fig. 10 and in Table 2 show a significantly lower dispersion of the peak response parameters in the Y-direction than in the X-direction. This is due to the fact that for each ground motion excitation, the three ground motion components were scaled uniformly based on $S_{XY}(T_1)$ and the fundamental building mode is in
Fig. 5. Elastic spectra (5% damped) of Y components of ground motions scaled to IM = $S_a(T_1)$ at 50% in 50 years hazard level. Note: the Y component used here is referred to as “longitudinal” by Somerville [31].

Fig. 6. Elastic spectra (5% damped) of X components of ground motions scaled to IM = $S_a(T_1)$ at 50% PE in 50 years hazard level. Note: the X component used here is referred to as “transversal” by Somerville [31].

Fig. 7. Elastic spectra (5% damped) of Z (vertical) components of ground motions scaled to IM = $S_a(T_1)$ at the 50% PE in 50 years hazard level.

Table 2
Coefficient-of-variations of peak response parameters (see Fig. 10) at 50% PE in 50 years hazard level.

<table>
<thead>
<tr>
<th></th>
<th>Peak floor disp., X</th>
<th>Peak floor disp., Y</th>
<th>Peak interstory drift., X</th>
<th>Peak interstory drift., Y</th>
<th>Peak floor abs. accel., X</th>
<th>Peak floor abs. accel., Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4-th floor</td>
<td>0.47</td>
<td>0.21</td>
<td>0.47</td>
<td>0.35</td>
<td>0.26</td>
<td>0.25</td>
</tr>
<tr>
<td>3-rd floor</td>
<td>0.50</td>
<td>0.25</td>
<td>0.47</td>
<td>0.22</td>
<td>0.31</td>
<td>0.34</td>
</tr>
<tr>
<td>2-nd floor</td>
<td>0.52</td>
<td>0.27</td>
<td>0.54</td>
<td>0.25</td>
<td>0.52</td>
<td>0.45</td>
</tr>
<tr>
<td>1-st floor</td>
<td>0.50</td>
<td>0.33</td>
<td>0.50</td>
<td>0.33</td>
<td>0.57</td>
<td>0.50</td>
</tr>
</tbody>
</table>
the Y-direction. Given the relatively high value of the initial fundamental period of the building ($T_1 = 1.07$ s), the peak floor absolute accelerations (PFAs) have a heightwise decreasing profile due to the deamplification with respect to the peak ground acceleration (PGA), typical of flexible structures.

At the 10% PE in 50 years hazard, the story maximum of the ensemble average of peak interstory drift ratios is nearly 2.5% in both X and Y-direction. At this level of interstory drift level, the columns have undergone significant inelastic deformations, as shown in Fig. 9. As compared to the lower hazard level, in this case the EDP dispersion generally decreases except for the EDP peak interstory drift ratio in the Y-direction at the lower floors (as shown in Table 3) where very high inelastic deformations are observed. The high dispersion of the EDP peak interstory drift ratio in the Y-direction influences the dispersion of the floor displacements too.

At the lower hazard level (50% PE in 50 years), the building response remains mainly elastic for the majority of the 20 ground motions, which results in a dispersion of the displacement EDPs in the Y-direction that is lower than that in the X-direction. This is due to the scaling criterion that enforces no dispersion around $S_{YX}(T_1)$ in the Y-direction and does not limit the dispersion of the spectral ordinates in the X-direction, namely $S_{XY}(T_2)$. On the other hand, at the higher hazard level (10% PE in 50 years), the response can be highly nonlinear, causing a significant shift in the modal periods. Thus, even in the Y-direction there is a high scatter of the spectral ordinates at the shifted fundamental period of vibration. This indicates that the scatter in the input propagates to a larger scatter in the EDPs at the higher hazard level, due to the large nonlinear demands on the structure at higher seismic intensities.

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**Fig. 8.** Building global and local responses to ground motion TO_trh02 (median GM for local response EDP = max curvature,Y in the 2nd story column labeled 2008) scaled to 50% PE in 50 years hazard level.

**Table 3**

<table>
<thead>
<tr>
<th>Roof</th>
<th>Peak floor disp._X</th>
<th>Peak floor disp._Y</th>
<th>Peak interstory drift._X</th>
<th>Peak interstory drift._Y</th>
<th>Peak floor abs. accel._X</th>
<th>Peak floor abs. accel._Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-th floor</td>
<td>0.39</td>
<td>0.49</td>
<td>0.32</td>
<td>0.36</td>
<td>0.14</td>
<td>0.21</td>
</tr>
<tr>
<td>3-rd floor</td>
<td>0.41</td>
<td>0.55</td>
<td>0.43</td>
<td>0.26</td>
<td>0.30</td>
<td>0.31</td>
</tr>
<tr>
<td>2-nd floor</td>
<td>0.43</td>
<td>0.67</td>
<td>0.47</td>
<td>0.58</td>
<td>0.36</td>
<td>0.33</td>
</tr>
<tr>
<td>1-st floor</td>
<td>0.47</td>
<td>0.76</td>
<td>0.47</td>
<td>0.76</td>
<td>0.37</td>
<td>0.40</td>
</tr>
</tbody>
</table>
Figs. 12 and 13 show the empirical cumulative distribution function (CDF) plots of the maximum (over all stories) peak interstory drift ratio (max PIDR) and the maximum (over all floors) PFA (max PFA) at the 50% and 10% PE in 50 years hazard level, respectively. These CDF plots provide the probability that a specified value of an EDP (here max PIDR and max PFA) has not been exceeded within a sample of 20 data points which represent the outcomes of the ensemble of nonlinear time-history analyses performed at a given seismic hazard level. The CDF plots in Figs. 12(a) and 13(a) provide four versions of the max PIDR, namely: (1) max PIDR in the X-direction (label X); (2) max PIDR in the Y-direction (label Y); (3) maximum peak rotational IDR (label R) defined as the maximum (over all floors) peak interstory rotation multiplied by half the diagonal of the rectangular floor; and (4) maximum peak resultant IDR (label V) which equals the square-root of the sum of squares of max PIDRs in X and Y-directions. All the aforementioned IDRs, including the rotational IDR and the resultant IDR, are illustrated in Fig. 2(b). Similarly, Figs. 12(b) and 13(b) provide the corresponding four versions of the max PFA with the rotational max PFA defined as the maximum (over all floors) peak floor angular acceleration multiplied by half the diagonal of the rectangular floor. Also, the coefficients of variation of the maximum (over all stories/floors) peak values of the above EDPs are reported in Table 4.

At the 50% PE in 50 years hazard level, the median value of the max PIDR in the X-direction is about 1.0%, while the median value of the max PIDR ratio V is about 1.4%, as shown in Fig. 12(a). The maximum (over all stories/floors) values of the rotational PIDRs and rotational PFAs (R) are much smaller than their counterparts due to the translational responses in the X and Y-directions (X, Y, and V) because the building has a relatively high torsional stiffness. Also, the max PIDR in the Y-direction (max PIDR_{Y}), where Y is the direction in which the ground motions are scaled to the same IM=Sa(T_1) is not systematically smaller or larger than the maximum PIDR in the X-direction (max PIDR_{X}). On the other hand, the max PFA in the X-direction (max PFA_{X}) is higher than the max PFA in the Y-direction (max PFA_{Y}) at both hazard levels. Finally, at the 10% PE in 50 years seismic hazard level (Fig. 13), the structure undergoes large inelastic deformations, mainly at the lower stories (first and second), with median max PIDRs in the X and Y-directions both in the 2–3% range (Fig. 13 (a)). The results in Table 4 show that the max PIDRs and maximum peak floor displacements exhibit higher dispersions in the X than in the Y-direction at 50% PE in 50 years hazard level, while the opposite happens at the 10% PE in 50 years hazard level. The dispersion in the max PFA is less sensitive to the response direction, and is lower at the 10% PE in 50 years hazard level.

5. Statistical correlation between IMs and EDPs for a given primary IM

The objective of the correlation study presented in this section is to test the hypothesis that the primary IM=S\_a(T\_1) is sufficient for
Fig. 10. Ensembles of peak response profiles in X and Y-directions for peak floor displacements, peak interstory drift ratios, and peak floor accelerations at 50% PE in 50 years hazard level (mean peak response profile in thick solid line).
Fig. 11. Ensembles of peak response profiles in X and Y-directions for peak floor displacements, peak interstory drift ratios, and peak floor accelerations at 10% in 50 years hazard level (mean peak response profile in thick solid line).
Fig. 12. Empirical CDF plots of the (a) max PIDR in the X and Y-directions (labels X and Y), max peak rotational IDR (label R), max peak resultant IDR (label V); and (b) max PFA in the X and Y-directions (labels X and Y), max peak rotational FA (label R), max peak resultant FA (label V) – 50% PE in 50 years hazard level.

Fig. 13. Empirical CDF plots of the (a) max PIDR in the X and Y-directions (labels X and Y), max peak rotational IDR (label R), max peak resultant IDR (label V); and (b) max PFA in the X and Y-directions (labels X and Y), max peak rotational FA (label R), max peak resultant FA (label V) – 10% PE in 50 years hazard level.

Table 4
Coefficient-of-variations of maximum (over all stories/floors) peak response parameters at 50% and 10% PE in 50 years hazard levels.

<table>
<thead>
<tr>
<th>Hazard level</th>
<th>Peak floor disp._X</th>
<th>Peak floor disp._Y</th>
<th>Peak interstory drift._X</th>
<th>Peak interstory drift._Y</th>
<th>Peak floor abs. accel._X</th>
<th>Peak floor abs. accel._Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>50% PE in 50 years</td>
<td>0.47</td>
<td>0.21</td>
<td>0.49</td>
<td>0.28</td>
<td>0.41</td>
<td>0.37</td>
</tr>
<tr>
<td>10% PE in 50 years</td>
<td>0.39</td>
<td>0.49</td>
<td>0.38</td>
<td>0.58</td>
<td>0.36</td>
<td>0.39</td>
</tr>
</tbody>
</table>
the 3-D nonlinear response analysis of the low-rise building considered herein. The primary IM is insufficient if an EDP conditional on $S_{Y}(T_{1})$ is strongly correlated to a secondary IM (strong correlation is assumed here to correspond to statistical correlation coefficients larger than 0.7). In other words, if the statistical correlation coefficient between the secondary IMs and an EDP conditional on $S_{Y}(T_{1})$ is large, then there is some information in the spectral representation of the ground motion records that can be used to better predict the EDPs. Therefore, it can be said that there is convincing evidence that the primary IM = $S_{Y}(T_{1})$ is not sufficient.

The probabilistic response analyses presented above were performed after scaling the 20 input ground motions to the IM = $S_{Y}(T_{1})$ at different hazard levels. All the Y-direction components of the elastic response spectra of the 20 scaled ground motion records intersect at the $S_{Y}(T_{1})$ (primary IM), while the spectral ordinates at different periods, or at all periods for the other two components, can have a significant scatter, as shown in Figs. 5–7. To investigate the effect of this scatter (variability) on the EDPs conditional on $S_{Y}(T_{1})$, a new set of IMs referred to as secondary IMs is introduced. The secondary IMs are defined as the spectral ordinates of the input ground motion at periods different from $T_{1}$ in the Y-direction and at all periods in the X-direction. In this study, the secondary IMs are selected as: (1) $S_{Y}(1.5T_{1})$ and $S_{Y}(2T_{1})$, which are the Y-direction spectral accelerations at shifted (elongated) fundamental mode periods representative of a structure experiencing inelastic response and stiffness degradation; (2) $S_{X}(T_{1})$, which is the spectral acceleration of the initial second mode in the Y-direction; (3) PGA$_{x}$ = $S_{X}(T = 0)$; and (4) $S_{X}(T_{1})$, which is the spectral acceleration of the initial first mode in the X-direction.

Figs. 14 and 15 show, on the left column, the probability of exceedance (or MAR of exceedance) curves obtained for the secondary IMs described above following the same procedure used to derive the curves in Fig. 4. Thus, the hazard curves obtained are derived starting from the three white-filled points on the three uniform hazard spectra at the secondary IM periods. These curves are used to determine the hazard level of the spectral acceleration at the secondary IM period for each of the 20 scaled ground motion records and for each period of the secondary IMs.

In each of the scatter plots shown on the right columns of Figs. 14 and 15, the values of the EDPs considered (max PFA$_{Y}$, max PIDR$_{X}$, and max PIDR$_{Y}$) are plotted against the corresponding values of the secondary IMs. The top X-axis indicates the PE in 50 years of the secondary IM defined in the legend of the bottom X-axis. The two groups of 20 black and white points correspond to the Monte Carlo simulation results of the building response performed based on the previously defined ensemble of 20 ground motion records scaled to the 50% PE in 50 years and 10% PE in 50 years hazard level, respectively, defined in terms of IM = $S_{Y}(T_{1})$. In each of the scatter plots, the statistical correlation coefficient between the considered EDPs and secondary IMs are also shown. For example, Fig. 14(b) shows that the max PIDR$_{X}$ has a moderate degree of statistical linear correlation ($\rho_{50} = 0.57$) with the secondary IM $S_{Y}(1.5T_{1})$ at the 50% PE in 50 years hazard level and a high correlation ($\rho_{10} = 0.91$) at the 10% PE in 50 years hazard level. Fig. 14(d) indicates that the max PIDR$_{X}$ has a moderate correlation ($\rho_{50} = 0.57$) with $S_{X}(2T_{1})$ at the 50% PE in 50 years hazard level and a high correlation ($\rho_{10} = 0.87$) at the 10% PE in 50 years hazard level. The statistical linear correlation of max PFA$_{Y}$ with the secondary IM $S_{Y}(T_{1})$ is found to be high at both the 50% PE in 50 years ($\rho_{50} = 0.88$) and 10% PE in 50 years ($\rho_{10} = 0.88$) hazard levels, as shown by the fairly linear distribution of points in Fig. 14(f).

The results in Fig. 14(b) and (d) clearly indicate that the max PIDR$_{X}$ (conditional on the primary IM = $S_{Y}(T_{1})$) is significantly correlated with the spectral ordinate of the Y component of the ground motion records at elongated fundamental periods ($T = 1.5T_{1}$ and $T = 2T_{1}$), representative of a structure that has undergone significant inelastic deformations. The degree of correlation increases with the hazard level of the primary IM (from 50% PE in 50 years to 10% PE in 50 years) as the level of inelastic demand on the structure increases. However, the correlation of the max PIDR$_{Y}$ with PGA$_{x}$ presented in Fig. 15(c) is high ($\rho_{50} = 0.70$) at the lower hazard level of $S_{Y}(T_{1})$, and it decreases ($\rho_{10} = 0.39$) at the higher hazard level of $S_{Y}(T_{1})$ of 10% PE in 50 years, indicating that PGA$_{x}$ is not a good predictor when nonlinear structural response is observed.

Figs. 14(f) and 15(b) show that the max PFA$_{Y}$ is highly correlated with the secondary IM = $S_{Y}(T_{1})$ (at the second mode frequency in the Y-direction) and with PGA$_{X}$, and that the degree of correlation ($\rho_{50} = \rho_{10} = 0.88$) with $S_{Y}(T_{1})$ is not affected by the hazard level of $S_{Y}(T_{1})$, while it decreases (from $\rho_{50} = 0.90$ to $\rho_{10} = 0.79$) with increasing hazard level of $S_{Y}(T_{1})$.

Fig. 15(e) shows the correlation of the max PIDR$_{X}$ (interstory drift ratio in the X-direction of the building) with the secondary IM $S_{X}(T_{2})$, where $T_{2}$ is the building first mode period in the X-direction. It was shown that by scaling uniformly the three components of the 20 ground motion records to match a specified value (Fig. 5) of $S_{Y}(T_{1})$ in the Y-direction, a relatively high variability results (Fig. 6) for spectral ordinate $S_{X}(T_{2})$, used here as secondary IM. The max PIDR$_{X}$ maintains a high degree of correlation with $S_{X}(T_{2})$ both at the lower and at the higher hazard level ($\rho_{50} = 0.77$ and $\rho_{10} = 0.72$, respectively) of $S_{Y}(T_{1})$ considered in this study.

It is worth noting that in Figs. 14(b), (d), (f) and 15(b), (c), (e), the hazard level of the secondary IM (as provided by the top X-axis) varies significantly from ground motion record to ground motion record and is very different from the hazard level of the primary IM = $S_{Y}(T_{1})$ with respect to which the ground motion records are conditioned.

It can be observed from Tables 2–4 that the c.o.v. of the floor displacements and interstory drift ratios in the Y-direction increase with increasing intensity of the ground motion, as expected and previously reported in the literature [16]. However, the c.o.v. of these same EDPs in the X-direction decrease with increasing hazard level. In the X-direction, due to inelastic behavior of the structure and corresponding period elongation, the first mode period in the X-direction ($T_{1}$) shifts towards the first mode period in the Y-direction ($T_{2}$), with a reduction in the dispersion of the response spectra (as shown in Fig. 6). Thus, in the X-direction, even though the c.o.v. of the floor displacements and interstory drift ratios would tend to increase due to the nonlinearity of the response, these c.o.v. are smaller for the 10% PE in 50 years than the 50% PE in 50 years hazard level.

Table 5 summarizes the relevant statistical correlation coefficients obtained between the EDPs and the secondary IMs at the two hazard levels of $S_{Y}(T_{1})$ considered. In general, it can be concluded that a moderate to high correlation exists between these EDPs and the secondary IMs. This is due to the nature of the scaling criterion based on the single scalar IM = $S_{Y}(T_{1})$. This illustrates the inadequacy of using a single scalar IM (here $S_{Y}(T_{1})$) to propagate the seismic input uncertainty to the uncertainty of the EDPs in the case of a structure, such as the low-rise building considered here, whose seismic response is strongly correlated to the spectral acceleration at periods significantly different from the initial fundamental period $T_{1}$. In this case, the primary IM = $S_{Y}(T_{1})$ lacks efficiency and sufficiency as defined by Luco and Cornell [8].

In summary, the significant degree of statistical correlation observed between some EDPs conditional on $S_{Y}(T_{1})$ and various secondary IMs (see Figs. 14 and 15, and Table 5) is convincing evidence that the primary IM = $S_{Y}(T_{1})$ is not sufficient.

6. Summary and conclusions

In this paper, part of the PEER probabilistic PBEE methodology was applied to an existing Italian four-story reinforced concrete building. This building was used as a testbed for evaluating the potential inadequacy/limitations of using a single/scalar IM in the PEER PBEE methodology when applied to a 3-D nonlinear model of a structure (subjected to three-directional earthquake ground motions) whose
Fig. 14. In left column, Figures (a), (c) and (e), probability of exceedance curves of the secondary IMs $S_{ay}(1.5T_1)$, $S_{ay}(2T_1)$ and $S_{ay}(T_4)$, respectively. In right column, Figures (b), (d) and (f), scatter plots and statistical correlation coefficients of the EDPs (max PIDR$_Y$ and max peak FA$_Y$) and the secondary IMs $S_{ay}(1.5T_1)$, $S_{ay}(2T_1)$ and $S_{ay}(T_4)$, respectively.
Fig. 15. In left column, Figures (a) and (d), probability of exceedance curves of the secondary IMs PGA\textsubscript{Y} and S\textsubscript{ax}(T\textsubscript{2}), respectively. In right column, Figures (b), (c) and (e) scatter plots and statistical correlation coefficients of the EDPs (max PFA\textsubscript{Y}, max PIDR\textsubscript{Y} and max PIDR\textsubscript{X}) and the secondary IMs PGA\textsubscript{Y} and S\textsubscript{ax}(T\textsubscript{2}), respectively.
response is strongly correlated to the spectral content of the seismic input away from its fundamental period $T_1$ and in different directions.

For the purpose of this study, this building structure was assumed to be located in the seismic environment corresponding to one of the sites of the PEER testbed program, namely the University of California Campus at Berkeley. Therefore, the probabilistic seismic hazard analysis results and the ensemble of 3-D ground motion records from this PEER testbed were used in this study. The ensemble of 20 three-directional ground motion records was conditioned (scaled) at two hazard levels (50% and 10% PE in 50 years) of the primary IM defined as $S_{XY}(T_1)$, the ordinate of the response spectrum of the ground motion $Y$ component at the fundamental period of the building $T_1$. This IM is widely accepted for predicting the seismic response of a two-dimensional structural model, especially for low-rise structures, for which the higher modes are expected to contribute weakly to the structural response. Based on the ensemble of ground motion records, a probabilistic response analysis was performed at the two hazard levels of $S_{XY}(T_1)$. The probabilistic response was characterized by a number of EDPS, including peak interstory drifts in X and Y-directions, peak floor relative displacements and absolute accelerations in X and Y-directions.

A statistical correlation analysis was performed between the EDPS (conditional on the primary IM) and the secondary IMs of the scaled ground motion records. The secondary IMs were defined as the spectral accelerations at elongated (due to damage) fundamental periods ($1.5T_1$ and $2T_1$) of the building and at periods of modes different than the fundamental mode. A number of these EDPS were found to be highly correlated to the secondary IMs conditional on the primary IM, with the hazard level of the secondary IMs varying significantly from record to record, differing from the hazard level of the primary IM.

The results obtained clearly illustrate that in the case presented here, the primary IM $S_{XY}(T_1)$, a commonly used single/scalar IM, lacks efficiency and sufficiency, two important attributes required for the IM in the PEER PBEE methodology. For 3-D structures such as that investigated herein, the violation of sufficiency will produce some erroneous/biased/unreliable results in the unconditional probabilistic response analysis which consists of convolving the probabilistic response conditional on the primary IM with the seismic hazard curve of the primary IM. The shortcoming of using the single scalar IM $S_{XY}(T_1)$ illustrated here, when using a 3-D model of a building structure subjected to a three-component seismic input, points to the need for a vector–valued IM in the PEER PBEE methodology for accurate/reliable probabilistic demand analysis of structures (e.g., building structures) whose seismic response is sensitive to the spectral ordinates at different periods (or period ranges) of the three components of the seismic input. The conclusions described above are expected to hold for more irregular 3-D structures, but further work is needed in this topic. It was not within the scope of this study to pursue the vector-based IMs. The study of simplified and accurate treatment of vector-based IMs, along with consistent ground motion selection and scaling, is a large endeavor on its own. However, as highlighted in the main conclusions of this paper, these are topics of ongoing and future research. The results obtained in this study imply several conclusions in terms of the future application of PBEE based on 3-D structural models and analyses for multi-component ground motions, which are discussed in the following paragraphs.

The only restrictive case in which the use of a single/scalar IM is strictly appropriate (i.e., satisfies the sufficiency condition) is the case in which the considered EDP is predominantly correlated to one component of the input ground motion and to the spectral ordinate at a single period of the structure. For all other cases, a single IM is not sufficient and a vector-valued IM is needed. Examples of such cases are given next: (1) The EDP is strongly correlated to one component only of the input ground motion and to the spectral ordinates of this component at several distinct periods (e.g., elongated periods due to inelastic structural response, higher mode effects); (2) The structure is irregular with strong torsional effects and the EDP is strongly correlated to each of the two horizontal components of the input ground motion and to the spectral ordinates of these two components at one or more periods of the torsional modes. Even if a single torsional mode contributes predominantly to an EDP, it is a two-component vector-IM problem with the two IMs being $S_{XY}(T_1)$ and $S_{XY}(T_2)$ where $X$ and $Y$ denote the two ground motion components; (3) In the general case, the EDPs depend on the spectral ordinates of multiple ground motion components at multiple periods, e.g., $S_{XY}(T_1), S_{XY}(T_2)$, and $S_{XY}(T_3)$.

All vector-valued IMs based on spectral ordinates of multi-component ground motions are rooted on the work of Baker and Jayaram [33] on (1) the correlation between the spectral ordinates of a given ground motion components at two different periods, and (2) the correlation between the spectral ordinates of two orthogonal ground motion components at a given period.

The treatment of a scalar EDPS assumed above and in most of the published work related to the PEER PBEE methodology can be extended to a vector-valued EDPS. The latter is of practical importance since common limit-state functions used to define structural performance depend on statistically correlated multiple EDPS, e.g., biaxial bending of columns, multiple criteria design problems.

Lastly, even for 2-D structural models and analyses, several authors concluded that a vector-valued IM can improve the accuracy of PBEE results for ground motions exhibiting near-fault effects [12,16]. These conclusions are expected to hold for 3-D analyses. Other ground motion characteristics, such as long duration and basin effects, are also expected to be well addressed by vector-valued IMs. While vector-valued IMs based on spectral ordinates of multi-component ground motions can make use of the work of Baker and Jayaram [33], the inclusion of specific ground motion characteristics (other than spectral ordinates) as components of a vector-valued IM is dependent on future developments of (1) IM-specific ground motion prediction equations, (2) statistical models for correlation between different component IMs, (3) robust ways of computing the seismic hazard surfaces and convolving them with the conditional probability distribution of the EDPS given a vector-valued IM, and (4) vector-valued IM consistent ground motion selection.

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