FULLY NONSTATIONARY ANALYTICAL EARTHQUAKE GROUND-MOTION MODEL

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ABSTRACT: A versatile, nonstationary stochastic ground-motion model accounting for the time variation of both intensity and frequency content typical of real earthquake ground motions is formulated and validated. An extension of the Thomson's spectrum estimation method is used to adaptively estimate the evolutionary power spectral density (PSD) function of the target ground acceleration record. The parameters of this continuous-time, analytical, stochastic earthquake model are determined by least-square fitting the analytical evolutionary PSD function of the model to the target evolutionary PSD function estimated. As application examples, the proposed model is applied to two actual earthquake records. In each case, model validation is obtained by comparing the second-order statistics of several traditional ground-motion parameters and the probabilistic linear-elastic response spectra simulated using the earthquake model with their deterministic counterparts characterizing the target record.

INTRODUCTION

A proper definition of the design ground-motion time history is a very important concern in structural earthquake engineering. To account for the uncertainties characterizing earthquake ground-motion time histories, several kinds of stochastic ground-motion models, stationary or nonstationary, have been developed and applied over the years. About every 10 years from 1970, comprehensive review papers (Liu 1970; Ahmadi 1979; Shinozuka and Deodatis 1988; Kozin 1988) examine and compare the stochastic earthquake ground-motion models available.

First, stationary white-noise ground-motion models were proposed (Housner 1947; Bycroft 1960). Accounting for local site properties and a dominant frequency in the ground motion, stationary nonwhite process models were suggested by Kanai (1957), Tajimi (1960), Housner and Jennings (1964), and Liu and Jhaveri (1969). Faravelli (1988) formulated a stationary ground-motion model with multimodal spectral density. However, stationary models fail to reproduce the time-varying intensity (or amplitude nonstationarity) typical of real earthquake ground-motion accelerograms. Therefore, a variety of time-modulating functions were introduced to produce various nonstationary ground-motion models. These models include the time-modulated harmonics (Bogdanoff et al. 1961), the filtered modulated white noise, the modulated filtered white noise, the modulated filtered Poisson process (Shinozuka and Sato 1967; Amin and Ang 1968), the modulated stationary process (Iyengar and Iyengar 1969), and the filtered modulated stationary process (Levy et al. 1971).

Furthermore, a time-varying frequency content is observed in actual earthquake records. This frequency nonstationarity depends on the epicentral distance, since it is due to the different arrival times of the P (primary or "push"), S (secondary or shear), and surface (Rayleigh and Love) waves that propagate at different velocities through the earth crust. These three types of waves vary significantly in frequency content. Thus, more complicated nonstationary ground-motion models were developed to represent both the amplitude and the frequency nonstationarity simultaneously. Saragoni and Hart (1972) proposed a fully nonstationary (with both amplitude and frequency nonstationarities) model by juxtaposing time segments of gamma-function-modulated filtered Gaussian white noise. Kubo and Penzien (1979) developed a nonstationary earthquake simulation model as the product of a constant intensity process having time-varying frequency content and a deterministic intensity function. Lin and Yong (1987) formulated evolutionary Kanai-Tajimi earthquake models as nonstationary random pulse trains and used Green's functions from one-dimensional wave propagation. Other researchers used both time- and frequency-modulating functions to construct a fully nonstationary earthquake model (Grigoriu et al. 1988; Yeh and Wen 1990). Der Kiureghian and Crempien (1989) defined an evolutionary earthquake model composed of individually modulated component stationary (band-limited white noise) processes. Fan and Ahmadi (1990) extended the original site-dependent, stationary, Kanai-Tajimi earthquake model to account for time-varying amplitude and frequency content. Papadimitriou (1990) produced a parsimonious nonstationary earthquake model by applying a second-order filter with slowly varying parameters to a time-modulated white noise. Conte et al. (1992) developed a time-varying autoregressive moving average (ARMA) model estimated from actual earthquake accelerograms using an iterative Kalman filtering scheme. Recently, several authors have developed fully nonstationary earthquake models using principles of geophysics and stochastic wave propagation (Deodatis et al. 1990; Zhang et al. 1991).

Most earthquake models have neglected the frequency nonstationarity for mathematical convenience in random vibration analysis and because it was believed that the temporal variation of the frequency content had no substantial effect on structural response. Several studies have shown that this nonstationarity in frequency content can have a significant effect on the response of both linear and nonlinear structures (Saragoni and Hart 1972; Yeh and Wen 1990; Papadimitriou 1990; Conte 1992).

In this paper, a new, versatile, fully nonstationary, stochastic earthquake model is proposed from the family of sigma-oscillatory processes. The model parameters are determined by adaptively least-square fitting the analytical time-varying (or evolutionary) power spectral density (PSD) function of the proposed model to the evolutionary PSD function estimated from the target actual earthquake accelerogram. This model-fitting procedure ensures that the earthquake model captures the time variation of both the intensity and the frequency content of the target earthquake record. A new approach, an extension of the Thomson's (1982) multiple window spectrum estimation method, is used to estimate the evolutionary PSD

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function of actual earthquake accelerograms. Thomson's method consists of an approximate solution to the fundamental equation of spectrum estimation, which is a linear Fredholm's integral equation of the first kind (Thomson 1982; Drosopoulous and Haykin 1992). Other methods exist to estimate the evolutionary PSD function from a single realization (time series) of a nonstationary process (Kameda 1975; Scherer et al. 1982; Spanos et al. 1987). Some attractive statistical properties of Thomson's (1982) spectral estimate are that: it is consistent, it has high resolution, its estimation capacity is high, and it is not hampered by the usual trade-off between bias (leakage) and variance. The proposed stochastic earthquake model is applied to two actual earthquake accelerograms. Artificial ground acceleration, velocity, and displacement time histories generated using the earthquake model are compared with the target time histories. The statistics of various traditional ground-motion parameters and the probabilistic linear response spectra (obtained from an ensemble of 100 artificial accelerograms simulated using the earthquake model) are compared to the target ground-motion parameters and linear-elastic response spectra, respectively. These comparisons serve to evaluate the ability of the proposed earthquake model to faithfully reproduce the multifold characteristics of actual earthquake ground motions.

FORMULATION OF STOCHASTIC EARTHQUAKE GROUND-MOTION MODEL

The evolutionary spectral analysis has been introduced by Priestley (1987). Priestley considers a class of nonstationary processes, the oscillatory processes, and defines the evolutionary (or time-varying) spectrum. Although this approach has proved to be widely applicable, it has some limitations. For example, the class of oscillatory processes is not closed with respect to the sum of independent elements, and the coherency of a bivariate oscillatory process turns out to be independent of time (Battaglia 1979). An attempt to get free of these limitations is presented by Battaglia, who introduces the concept of sigma-oscillatory processes and defines an evolutionary spectral analysis (time-frequency distribution analysis with a physical frequency parameter) for this kind of processes. A stochastic process is termed sigma-oscillatory if it is defined as the sum of a finite number of pairwise (statistically) independent oscillatory processes. It can be shown that the family of sigma-oscillatory processes is closed with respect to the sum of independent elements and the coherency function of a bivariate sigma-oscillatory process is time-dependent. Here, a nonstationary, stochastic earthquake ground-motion model based on the theory of sigma-oscillatory processes is proposed.

Consider a sigma-oscillatory process, $Y(t)$, defined as

$$Y(t) = \sum_{k=1}^{p} X_k(t) \tag{1}$$

in which the component processes $[X_k(t), k = 1, 2, \ldots, p]$ are oscillatory processes admitting the spectral representation

$$X_k(t) = \int_{-\infty}^{\infty} A_k(t, \omega)e^{j\omega t}dZ_k(\omega) \tag{2}$$

In (2), $j = \sqrt{-1}$; $A_k(t, \omega)$ = frequency-time (deterministic) modulating function; and quantities $[dZ_k(\omega)]$ = zero-mean, mutually independent, orthogonal increment processes having the properties

$$E[dZ_k(\omega)] = 0, \; k = 1, 2, \ldots, p \tag{3}$$

$$E[dZ_j^*(\omega_1)dZ_k(\omega_2)] = \delta(j - k)\delta(\omega_1 - \omega_2)\Phi_{Z_kZ_k}(\omega_1)\, d\omega_1\, d\omega_2 \tag{4}$$

in which $j, k = 1, 2, \ldots, p$; $E[\;]$ = ensemble-average or expectation operator; $\delta(\;)$ = Dirac delta function; and $*$ = complex conjugate. The spectral representation in (2) can be physically interpreted as the limit of a "sum" of sine waves with increasing frequencies and time-varying random amplitudes $[A_k(t, \omega)dZ_k(\omega)]$. Each component process $X_k(t)$ of the sigma-oscillatory process $Y(t)$ has the following evolutionary spectrum:

$$\Phi_{X_kX_k}(t, \omega) = |A_k(t, \omega)|^2\Phi_{Z_kZ_k}(\omega) \tag{5}$$

with respect to the oscillatory family of functions, $\mathcal{F}_k = [A_k(t, \omega)e^{j\omega t}]$, which should be viewed as functions of ω indexed by t. For simplicity, it is assumed that each spectrum is absolutely continuous with respect to ω. According to Priestley's (1987) definition of oscillatory processes, the modulating function $A_k(t, \omega)$ (viewed as a function of t for each ω) must be such that the modulus of its Fourier transform $H_k(\theta, \omega)$ has an absolute maximum at the origin (i.e., $\theta = 0$) and

$$A_k(t, \omega) = \int_{-\infty}^{\infty} e^{j\theta t}H_k(\theta, \omega)\, d\theta \tag{6}$$

The mean-square function of the sigma-oscillatory process $Y(t)$ defined in (1) is

$$E[|Y(t)|^2] = \sum_{k=1}^{p} E[|X_k(t)|^2] = \int_{-\infty}^{\infty} \sum_{k=1}^{p} [|A_k(t, \omega)|^2\Phi_{Z_kZ_k}(\omega)]\, d\omega \tag{7}$$

which gives a decomposition over frequency of the "total energy" of $Y(t)$ at time t measured by the variance. Therefore, the evolutionary (time-varying) power spectrum of $Y(t)$ can be meaningfully defined with respect to the oscillatory family of functions $\mathcal{F}_Y = \cup_{k=1}^{p} \mathcal{F}_k$ by

$$\Phi_{YY}(t, \omega) = \sum_{k=1}^{p} |A_k(t, \omega)|^2\Phi_{Z_kZ_k}(\omega) \tag{8}$$

The sum of two independent sigma-oscillatory processes remains a sigma-oscillatory process whose evolutionary spectrum is the sum of the evolutionary spectra of the two individual processes. Moreover, the characteristic width of the family $\mathcal{F}_Y$, and the characteristic width of the process $Y(t)$ are defined as

$$B_{\mathcal{F}_Y} = \min_{1\le k\le p} B_{\mathcal{F}_k}; \quad B_Y = \min_{1\le k\le p} B_{X_k} \tag{9a,b}$$

where

$$B_{\mathcal{F}_k} = 1/\sup_{\omega} \left[\int_{-\infty}^{\infty} |\theta||H_k(\theta, \omega)|\, d\theta \Big/ \int_{-\infty}^{\infty} |H_k(\theta, \omega)|\, d\theta\right]$$

is the characteristic width of $\mathcal{F}_k$; and B_{X_k} = characteristic width of the component process $X_k(t)$ defined by $B_{X_k} = \sup_{\mathcal{F}_k\in\mathcal{G}_k} B_{\mathcal{F}_k}$, in which $\mathcal{G}_k$ = class of families $\mathcal{F}_k$ with respect to which $X_k(t)$ admits the spectral representation in (2). If the process $X_k(t)$ is stationary, B_{X_k} is infinite. If B_{X_k} is finite, the nonstationary process $X_k(t)$ is termed semi-stationary. The characteristic width is a measure of the nonstationarity of a process; roughly speaking, $2\pi B_{X_k}$ or $2\pi B_Y$ may be interpreted as the maximum time interval over which $X_k(t)$ or $Y(t)$ can be treated as approximately stationary.

In this paper, a fully nonstationary, stochastic earthquake ground acceleration model, $\ddot{U}_g(t)$, is defined as a sum of zero-mean, independent, uniformly modulated Gaussian processes. Each uniformly modulated process consists of the product of a deterministic time-modulating function, $A_k(t)$, and a stationary Gaussian process, $S_k(t)$. Thus, the proposed stochastic earthquake model is a particular sigma-oscillatory Gaussian process defined as

$$\ddot{U}_g(t) = \sum_{k=1}^{p} X_k(t) = \sum_{k=1}^{p} A_k(t)S_k(t) \quad (10)$$

Furthermore, the modified gamma function is used as a time-modulating function, i.e.

$$A_k(t) = \alpha_k(t - \zeta_k)^{\beta_k} e^{-\gamma_k(t-\zeta_k)} H(t - \zeta_k) \quad (11)$$

where α_k and γ_k = positive constants; β_k = a positive integer; ζ_k = the "arrival time" of the kth subprocess, $X_k(t)$; and $H(t)$ = unit step function. The kth zero-mean stationary Gaussian process, $S_k(t)$, is characterized by its autocorrelation function

$$R_{S_kS_k}(\tau) = e^{-\nu_k|\tau|} \cos(\eta_k \tau) \quad (12)$$

and its power spectral density function

$$\Phi_{S_kS_k}(\omega) = \frac{\nu_k}{2\pi}\left[\frac{1}{\nu_k^2 + (\omega + \eta_k)^2} + \frac{1}{\nu_k^2 + (\omega - \eta_k)^2}\right] \quad (13)$$

in which ν_k and η_k = two free parameters representing the frequency bandwidth and predominant (or central) frequency of the process $S_k(t)$, respectively. The stationary Gaussian process $S_k(t)$ possesses a physical interpretation. It can be viewed as the linear combination of the displacement and velocity responses of a second-order single-degree-of-freedom (SDOF) oscillator subjected to two statistically independent Gaussian white noise processes. A set of linear combination coefficients can be found such that the autocorrelation function of the combined process coincides with (12). The stationary processes, $[S_k(t), k = 1, 2, \ldots, p]$, are normalized in order to have a unit variance. Since the ground acceleration is modeled as a sigma-oscillatory process, according to (7), the mean-square ground acceleration is given by

$$E[|\ddot{U}_g(t)|^2] = \int_{-\infty}^{\infty} \sum_{k=1}^{p} |A_k(t)|^2 \Phi_{S_kS_k}(\omega)\, d\omega = \sum_{k=1}^{p} |A_k(t)|^2 \quad (14)$$

where

$$\int_{-\infty}^{\infty} \Phi_{S_kS_k}(\omega)\, d\omega = R_{S_kS_k}(\tau)|_{\tau=0} = E[|S_k(t)|^2] = 1 \quad (15)$$

From (8), the evolutionary PSD function of $\ddot{U}_g(t)$ is

$$\Phi_{\ddot{U}_g\ddot{U}_g}(t, \omega) = \sum_{k=1}^{p} |A_k(t)|^2 \Phi_{S_kS_k}(\omega) \quad (16)$$

The ground acceleration process, $\ddot{U}_g(t)$, is not separable although its component processes are individually separable (i.e., uniformly modulated). Each uniformly modulated component process, $X_k(t)$, is characterized by a unimodal PSD function in the frequency domain and a unimodal mean-square function in the time domain. Therefore, each component process captures the complex time-frequency distribution of the earthquake ground acceleration in a local time-frequency region. In other words, the proposed earthquake model views the earthquake ground motion process as the superposition of component processes described by their own arrival time, frequency content, and time-intensity function. Each of these component processes can represent a specific group of seismic waves. As a particular case, the subprocesses of a three-component model could represent the P, S, and surface wave groups present in earthquake accelerograms. However, the proposed model allows for an arbitrary number of subprocesses, which is determined by the level of detail desired in capturing the time-frequency distribution of the target accelerograms.

The aforementioned stochastic earthquake ground-motion is useful to simulate potential, future earthquake records at a site for which previous earthquake accelerograms are available. However, this model can also be used when no accelerograms are available. In this case, either (1) a target accelerogram is selected from another site that is similar in terms of earthquake source mechanism, wave propagation, and local soil conditions; or (2) the evolutionary PSD of the underlying earthquake process is generated numerically using newly available geophysical, stochastic earthquake models (Deodatis et al. 1990; Zhang et al. 1991). Even in the case where a geophysical, stochastic earthquake model is needed to provide the evolutionary PSD, the proposed model is useful to perform analytical random vibration studies. The earthquake model is identified such that it captures, in an optimum sense, the estimated or a priori generated evolutionary PSD of the ground acceleration. This paper focuses on the case for which an earthquake record is available.

ESTIMATION OF MODEL PARAMETERS

The parameters of the earthquake ground acceleration model, $\ddot{U}_g(t)$, are estimated such that the analytical evolutionary PSD function, $\Phi_{\ddot{U}_g\ddot{U}_g}(t, \omega)$, given in (16) best fits, in the least-square sense, the evolutionary PSD function of the target earthquake accelerogram estimated using the short-time Thomson's multiple-window method, $\hat{\Phi}_{\ddot{U}_g\ddot{U}_g}(t, \omega)$, which will be explained later. The estimated evolutionary PSD function consists of the discrete set of data $[\hat{\Phi}_{\ddot{U}_g\ddot{U}_g}(t_i, \omega_j), i = 1, 2, \ldots, N_t; j = 1, 2, \ldots, N_\omega]$. By (11), (13), and (16), the analytical evolutionary power spectrum is a function of the parameter vector $\Theta = [\theta_1, \theta_2, \ldots, \theta_{6p}]^T$ whose components are (α_k, β_k, γ_k, ζ_k, ν_k, η_k, $k = 1, 2, \ldots, p$). The error or objective function is defined by

$$J(\Theta) = \frac{1}{2}\sum_{i=1}^{N_t}\sum_{j=1}^{N_\omega} [\Phi_{\ddot{U}_g\ddot{U}_g}(t_i, \omega_j, \Theta) - \hat{\Phi}_{\ddot{U}_g\ddot{U}_g}(t_i, \omega_j)]^2 \quad (17)$$

The objective function is minimized with respect to the parameter vector Θ, which is subjected to simple bound constraints, i.e., all the parameters should be positive except for ζ_k. The existence of explicit closed-form solutions for the time-varying PSD functions and auto/cross-correlation functions of the response of linear-elastic structures excited by the present earthquake model requires the β_k parameters to be integers (Conte and Peng 1996). It can be shown that, under the foregoing simple bound constraints, the objective function and its partial derivatives with respect to the parameter vector Θ are continuous functions of Θ, which is a desirable property for numerical minimization of the objective function.

The first step in the model identification procedure consists of choosing the number, p, of independent component processes to be included in the earthquake model. As p and the number of free parameters (=$6p$) increase, the model can potentially fit better the estimated data and further reduce the value of the objective function. However, a model with too many degrees of freedom can lead to singular convergence. This can be overcome by using more estimated data, thus increasing the cost of computation. Therefore, there is a trade-off between accuracy and efficiency of the model. The principle of model parsimony must be applied, namely that the simplest model which is accurate enough must be sought. The experience gained by applying the model to several actual earthquake records indicates that an accurate description of the time-frequency distribution of the ground acceleration usually requires a value of p between 10 and 20. This range of values of p leads to a rather large number of model parameters (=$6p$). The main objective of the research project that contains this study is to investigate the influence of the frequency nonstationarity of earthquake ground motions on structural response. Therefore, initially it is essential to capture accurately the time-frequency distribution of the ground motion. Further studies will indicate the level of detail of the evolutionary PSD to which structural response is significantly sensitive. Then, based on these results, model parsimony will be sought and correlation studies will be performed between the earthquake model parameters and geophysical parameters such as earthquake magnitude, epicentral distance, local soil conditions, etc. These relationships will be useful to simulate artificial earthquake records at sites where no accelerograms are available.

The next step consists of selecting initial parameter values to start the search algorithm. Different sets of initial parameter values may lead to different converged results. Therefore, a smart choice of the initial parameter vector is very important to reach the desired minimum of the objective function. Here, a procedure is outlined, which determines appropriate initial parameter values. In the contour plot of the estimated time-varying PSD function, many local maxima appear over the time-frequency domain considered. This whole domain is divided into several subdomains each of which surrounds one to three local maxima. Since each independent component process is able to capture a single local maximum only, the estimated data in each time-frequency subdomain can be best fitted using, at most, three component processes. The optimum parameter values for the component processes attached to each subdomain are determined by solving a local optimization problem of reduced size. After all the local optimization problems are solved, the real-valued (β_k) parameters are permanently reset to their nearest integer. Numerical experience has shown that the locally optimized, integer-valued β_k parameters are usually not affected by the subsequent global parameter optimization. Therefore, they are not included in the global parameter vector. All the other subdomain optimum parameters are grouped to form the initial global parameter vector. This procedure to initialize the global parameter vector has proven successful in the application examples. Finally, the global constrained minimization problem is solved by applying the adaptive nonlinear least-squares algorithm named "NL2SOL" (Dennis et al. 1981).

SHORT-TIME THOMSON'S MULTIPLE-WINDOW SPECTRUM ESTIMATION

The usual assumption made in estimating the characteristics of nonstationary processes is that of local stationarity. The classical nonparametric spectrum estimation method, called short-time Fourier transform (STFT), is a moving time-window technique. Estimates of the evolutionary power spectrum of a sigma-oscillatory process can be obtained in exactly the same form as in the oscillatory case, i.e., by means of a linear transformation. This implies that the STFT is an admissible method provided the "width" of the moving time window is smaller than the characteristic width, $2\pi B_{\ddot{U}_g}$, of the underlying target nonstationary process, $\ddot{U}_g(t)$. Therefore, the moving-window spectrum estimation method can be applied to estimate the evolutionary power spectrum of a nonstationary process.

Thomson (1982) introduced the multiple-window spectral estimation method for stationary data. Thomson's method has the advantages of being consistent, having high resolution and high estimation capacity, and of not being hampered by the usual trade-off between leakage and variance. Here, this multiple-window method is extended to estimate the time-varying power spectrum of a nonstationary time series (i.e., earthquake ground acceleration). This extended method is called the short-time Thomson's multiple-window spectrum estimation method and is briefly outlined as follows.

First, a time-moving window of size $N\cdot\Delta t$, $[w(n), n = 0, 1, \ldots, N-1]$, where Δt denotes the sampling time interval, is used to extract the local time series centered at time t_i, from the target earthquake ground acceleration record, $[\ddot{U}_g(t_i), t_i = i\cdot\Delta t, i = 0, 1, \ldots, M-1]$. Thus, the local time series centered at time t_i is

$$\{S(t_i, n) = \ddot{U}_g[t_{i+n-(N-1)/2}]w(n),\ n = 0, 1, \ldots, N-1\}\ \text{(for } N \text{ odd)} \tag{18a}$$

$$[S(t_i, n) = \ddot{U}_g(t_{i+n-N/2})w(n),\ n = 0, 1, \ldots, N-1]\ \text{(for } N \text{ even)} \tag{18b}$$

in which the time-moving window is normalized such that $\sum_{n=0}^{N-1} w^2(n) = 1$.

Then, the local time series $[S(t_i, n)]$ is projected onto several (K) leakage-resistant orthogonal windows that are the discrete prolate spheroidal sequences (DPSSs) denoted by $[v_k(n), k = 0, 1, \ldots, K-1; n = 0, 1, \ldots, N-1]$. It can be shown (Drosopoulos and Haykin 1992) that these DPSSs are the eigenvectors of the Toeplitz matrix $\mathbf{T}$, i.e.

$$\mathbf{T}\mathbf{V}_k = \lambda_k \mathbf{V}_k \tag{19}$$

in which the components of the $\mathbf{T}$ matrix are given by $T_{ij} = \{\sin[2\pi W(i-j)]\}/[\pi(i-j)]$; $\mathbf{V}_k = [v_k(0), v_k(1), \ldots, v_k(N-1)]^T$ = kth eigenvector of $\mathbf{T}$; λ_k = kth eigenvalue; and $W = (K)/(2N)$ = bandwidth of the discrete-time Fourier transforms of the first K DPSSs. The centered discrete-time Fourier transform of the kth DPSS is also known as the kth discrete prolate spheroidal wave function (DPSWF) defined as

$$\varphi_k(f) = \varepsilon_k \sum_{n=0}^{N-1} v_k[n] e^{j2\pi f[n-(N-1)/2]} \tag{20}$$

where $\varepsilon_k = 1$, for k even; and j, for k odd is introduced so that $\varphi_k(f)$ is a real-valued function. The DPSWFs appear in the solution of a particular form of the Sturm-Liouville problem called the prolate spheroidal differential equation of zero order (Flammer 1957). It can be shown that they satisfy the following two homogeneous integral equations:

$$\int_{-W}^{W} \frac{\sin N\pi(f-\nu)}{\sin \pi(f-\nu)} \varphi_k(\nu)\, d\nu = \lambda_k \varphi_k(f) \tag{21}$$

$$\int_{-1/2}^{1/2} \frac{\sin N\pi(f-\nu)}{\sin \pi(f-\nu)} \varphi_k(\nu)\, d\nu = \varphi_k(f) \tag{22}$$

in which $[\sin N\pi(f-\nu)]/[\sin \pi(f-\nu)]$ is referred to as the Dirichlet kernel denoted by $D_N(f-\nu)$.

The DPSSs and DPSWFs are normalized and the DPSWFs satisfy the following orthogonal properties in both the $[-W, W]$ and $[-(1/2), (1/2)]$ domains:

$$\int_{-W}^{W} \varphi_j(f)\varphi_k(f)\, df = \lambda_k \delta(j-k) \tag{23}$$

$$\int_{-1/2}^{1/2} \varphi_j(f)\varphi_k(f)\, df = \delta(j-k) \tag{24}$$

By (23) and (24), the kth eigenvalue can be expressed as

$$\lambda_k = \frac{\int_{-W}^{W} \varphi_k(f)\varphi_k(f)\, df}{\int_{-1/2}^{1/2} \varphi_k(f)\varphi_k(f)\, df} \tag{25}$$

and therefore represents the fractional energy concentrated in the domain $(-W, W)$ for the kth DPSWF. The eigenvalues are positive ($\mathbf{T}$ is symmetric, positive-definite), less than one, and ordered such that $1 > \lambda_0 > \lambda_1 > \cdots > \lambda_{N-1} > 0$. The first K $(=2NW)$ eigenvalues are close to unity and thus the corresponding DPSS windows are useful for minimizing spectral leakage. However, spectral leakage resistance becomes progressively poorer as the order of the DPSS increases. For this reason, only the first K DPSS windows are used, since they are both frequency band-limited and time-limited. These last two properties are very useful in spectrum estimation (Slepian 1978). Therefore, corresponding to each centered time t_i, there are K projected local time series, which are $[S(t_i, n)v_k(n), n = 0, 1, \ldots, N - 1]$, $k = 0, 1, \ldots, K - 1$. The statistical information of $[S(t_i, n)]$ discarded by the first DPSS window is partially recovered by the second DPSS window, the information discarded by the first two DPSS windows is partially retrieved by the third DPSS window, and so on.

By taking the discrete Fourier transform (DFT) of these projected local-time series, several (K) "prolate local-time eigenspectra" estimates are produced as

$$\hat{S}_k[t_i, \omega_j] = \left| \sum_{n=0}^{N-1} S(t_i, n)v_k(n)e^{-j\omega_j[n-(N-1/2)]} \right|^2, \quad k = 0, 1, \ldots, K - 1 \tag{26}$$

in which $\omega_j = (2\pi j)/(N \cdot \Delta t)$, $j = 0, 1, \ldots, N - 1$; $i = 0, 1, \ldots, M - 1$. These local-time eigenspectra are combined to form a local-time spectrum estimate, $\hat{S}(t_i, \omega_j)$, by introducing time-frequency-dependent weighting functions, $[d_k(t_i, \omega_j)]$, to reduce bias from spectral leakage. These weighting functions are determined adaptively. Then, the adaptive short-time Thomson's spectrum estimation reduces to the following two equations, which must be solved iteratively:

$$\hat{S}(t_i, \omega_j) = \frac{\sum_{k=0}^{K-1} |d_k(t_i, \omega_j)|^2 \hat{S}_k(t_i, \omega_j)}{\sum_{k=0}^{K-1} |d_k(t_i, \omega_j)|^2} \tag{27}$$

$$d_k(t_i, \omega_j) = \frac{\sqrt{\lambda_k}\hat{S}(t_i, \omega_j)}{\lambda_k \hat{S}(t_i, \omega_j) + (1 - \lambda_k)\sigma_{t_i}^2} \tag{28}$$

where $\sigma_{t_i}^2 = \sum_{n=0}^{N-1} S^2(t_i, n)$ = mean-square estimate for $\ddot{U}_g(t_i)$.

Similar to Priestley's method for estimating the evolutionary power spectrum, a time average window $[w_T(n), n = 1, 2, \ldots, N_T - 1]$ is introduced to reduce the sampling fluctuation along the time axis and produce a power spectrum estimate varying slowly in time. Thus, the short-time Thomson's multiple-window spectrum estimate, $\hat{\Phi}_{\ddot{U}_g\ddot{U}_g}(t_i, \omega_j)$, of the target underlying earthquake process $\ddot{U}_g(t)$ is

$$\hat{\Phi}_{\ddot{U}_g\ddot{U}_g}(t_i, \omega_j) = \sum_{n=0}^{N_T-1} \hat{S}[t_{i+n-(N_T-1)/2}, \omega_j]w_T(n) \tag{29}$$

Finally, the foregoing estimate $\hat{\Phi}_{\ddot{U}_g\ddot{U}_g}(t_i, \omega_j)$ is scaled to satisfy the local variance of $\ddot{U}_g(t)$, i.e.

$$\sum_{j=0}^{N-1} \hat{\Phi}_{\ddot{U}_g\ddot{U}_g}(t_i, \omega_j) \cdot \Delta\omega_j = \sigma_{t_i}^2 \tag{30}$$

APPLICATION EXAMPLES AND MODEL VALIDATION

The stochastic earthquake model proposed here is applied to two real earthquake records having different nonstationarity characteristics. The first record corresponds to the S00E (N-S) component of the Imperial Valley earthquake of May 18, 1940, recorded at the El Centro site. The second record is the N00W (N-S) component of the San Fernando earthquake of February 9, 1971, recorded at the Orion Boulevard site.

El Centro 1940, North-South Component

Fig. 1 represents the estimated time-varying PSD function, $\hat{\Phi}_{\ddot{U}_g\ddot{U}_g}(t, \omega)$, for the El Centro 1940 earthquake ground acceleration record, based on the short-time Thomson's multiple-window spectrum estimation method. A Hanning window of size 4 s (N = 200, Δt = 0.02 s) is chosen as time-moving window to extract the local time series. The first two DPSSs (K = 2) are selected as the projecting windows and, therefore, for each local time series, two eigenspectra are estimated, namely $S_1(t, \omega)$ and $S_2(t, \omega)$. From (27)–(30) and using a time-average Hanning window of size N_T = 200, the time-varying PSD function is estimated adaptively.

By the adaptive nonlinear least-squares algorithm, the parameters of the sigma-oscillatory process model composed of 21 independent component modulated oscillatory processes are estimated and reported in Table 1. Note that $2\pi B_{\mathcal{F}_k}$ is a dependent parameter representing the characteristic width of the kth component process and the parameter β_k is pre-estimated during the parameter initialization procedure described earlier. Therefore, the final number of free parameters to be estimated is 105 (=21 × 5). Fig. 2 portrays the analytical time-varying PSD function, $\Phi_{\ddot{U}_g\ddot{U}_g}(t, \omega)$, of the identified nonstationary stochastic model.

Fig. 3(a) shows both the analytical mean-square ground acceleration function from the identified earthquake model and the one directly estimated from the target earthquake record using a running average. Fig. 3(b) represents both the modeled and the estimated global PSD function, $S_T(\omega)$, defined as

$$S_T(\omega) = \int_0^{t_d} \Phi_{\ddot{U}_g\ddot{U}_g}(\tau, \omega)\, d\tau \tag{31}$$

in which t_d = duration of the target ground acceleration record considered, e.g., for the El Centro 1940 record, t_d = 35 s. Figs.

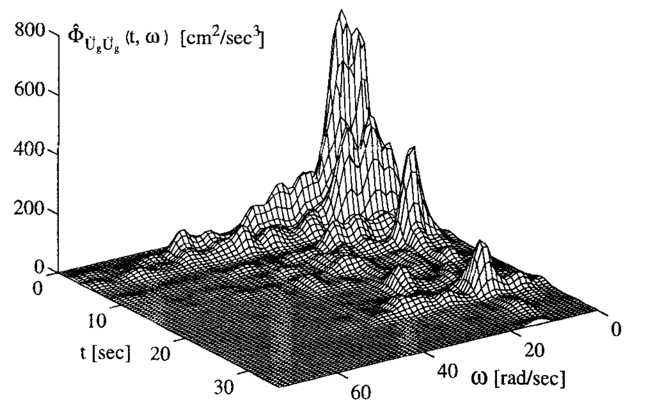

FIG. 1. Estimated Time-Varying Power Spectral Density Function for El Centro 1940 Earthquake Ground Acceleration

TABLE 1. Estimated Parameters of Ground Acceleration Model for El Centro 1940 Earthquake Record

k (1)	α_k ( ) (2)	β_k ( ) (3)	γ_k ( ) (4)	ζ_k (s) (5)	ν_k ( ) (6)	η_k (rad/s) (7)	$2\pi B_{\mathcal{F}_k}$ (s) (8)
1	37.2434	8	2.7283	−0.5918	1.4553	6.7603	7.3696
2	104.0241	8	2.9549	−0.9857	2.4877	11.0857	6.8043
3	31.9989	8	2.6272	1.7543	3.3024	7.3688	7.6531
4	43.8375	9	3.1961	1.6860	2.1968	13.5917	6.7551
5	33.1958	9	3.1763	−0.0781	3.1241	14.3825	6.7972
6	41.3111	9	3.1214	−0.7096	6.7335	25.1532	6.9168
7	4.2234	10	2.9904	−0.9464	2.6905	48.0612	7.6840
8	19.9802	6	1.8950	1.4020	7.2086	37.6163	8.8420
9	2.4884	10	2.6766	5.3123	6.1101	19.4612	8.5851
10	24.1474	10	3.3493	8.8564	1.9862	9.040	6.8608
11	2.5916	2	0.2240	3.2558	2.4201	9.3381	28.0509
12	2.2733	3	0.5285	16.2065	1.5244	14.1067	18.6763
13	24.2732	3	1.0361	17.5331	1.7141	24.0444	9.5254
14	41.0734	2	0.7511	22.3717	5.9541	27.7953	8.3648
15	1.3697	10	2.5936	21.6830	1.9362	12.9198	8.8597
16	15.4646	2	0.7044	27.2979	1.7897	12.0205	8.9205
17	0.0174	10	1.8451	−2.4168	4.9373	98.6280	12.4538
18	2.9646	10	3.1137	1.5751	1.9726	61.8316	7.3798
19	0.0007	10	1.3686	2.5173	3.2479	43.9075	16.7901
20	0.8092	4	0.5969	6.4396	3.6749	26.3365	21.0531
21	16.7115	2	0.7294	12.4930	1.7075	37.1139	8.6137

1–3 indicate that the identified analytical earthquake model captures very well the time-frequency distribution of the El Centro 1940 ground acceleration record.

It is worth recalling that the short-time Thomson's multiple-window spectrum estimation method assumes local stationarity of the target process. This assumption is satisfied if the effective width, $2\pi B_w$, of the time-moving window, $w(u)$, is smaller than the characteristic bandwidth, $2\pi B_{\ddot{U}_g}$, of the target nonstationary process. The effective width $2\pi B_w$ is defined as (Priestley 1987)

$$2\pi B_w = 2\pi \int_{-\infty}^{\infty} |u| w(u) \, du \tag{32}$$

For the Hanning window of size 4 s used in the present example, this effective width is

$$2\pi B_w = \frac{\pi}{\sqrt{3\pi}} \int_{-2}^{2} |u| \left\{1 - \cos\left[\frac{\pi}{2}(u + 2)\right]\right\} du = 2.434 \text{ s} \tag{33}$$

From (9) and Table 1, the characteristic bandwidth of the analytical ground acceleration model is $2\pi B_{\ddot{U}_g} = \min_{1 \le k \le 21} 2\pi B_{\mathcal{F}_K} = 6.7551$ s where

$$B_{\mathcal{F}_K} = \frac{\int_{-\infty}^{\infty} |A_k(\theta)| \, d\theta}{\int_{-\infty}^{\infty} |\theta| |A_k(\theta)| \, d\theta} = \frac{(\beta_k - 1)\Gamma\left(\frac{\beta_k}{2}\right)\sqrt{\pi}}{2\gamma_k \Gamma\left(\frac{\beta_k}{2} + \frac{1}{2}\right)} \tag{34}$$

$$A_k(\theta) = \int_0^{\infty} A_k(t) e^{j\theta t} \, dt = \frac{\alpha_k \beta_k!}{(j\theta + \gamma_k)^{\beta_k + 1}} \tag{35}$$

in which $\Gamma(\ )$ = gamma function. Therefore, the assumption of local stationarity is satisfied for this example.

The first level of model validation is performed by simulating a sample of 100 artificial accelerograms from the identified earthquake model and computing the second-order statistics (i.e., mean and standard deviation) of 10 ground-motion parameters traditionally used to characterize earthquake intensity. These ground motion parameters are:

- Peak ground acceleration (PGA), PG velocity (PGV), and PG displacement (PGD).
- Ratios of peak ground velocity and displacement to peak ground acceleration: PGV/PGA and PGD/PGA.
- Root-mean-square acceleration (RMSA), RMS velocity (RMSV), and RMS displacement (RMSD).
- Arias intensity (AI) defined as $\text{AI} = \pi/(2g) \int_0^{t_d} \ddot{U}_g^2(t) \, dt$ in which g is the acceleration of gravity.
- Housner spectral intensity (SI_ξ) defined as $\text{SI}_\xi = \int_{0.1}^{2.5} \text{PSV}(\xi, T) \, dT$ where $\text{PSV}(\xi, T)$ denotes the pseudo-spectral velocity for a given real or artificial earthquake record and for a SDOF oscillator of damping ratio ξ and undamped natural period T.

In simulating the analytical ground-motion model, the component processes are generated independently using the spectral representation method (Shinozuka and Jan 1972) and combined together to form one realization of the ground acceleration process. The artificial ground motions simulated are baseline-corrected in the frequency domain by using a simple rectangular high-pass filter with a cutoff frequency of 0.10

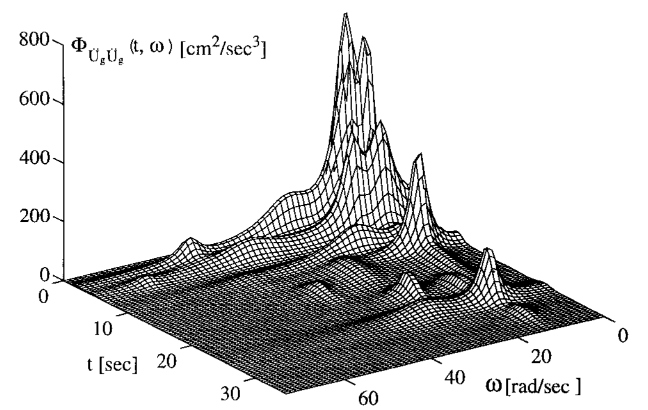

FIG. 2. Analytical Time-Varying Power Spectral Density Function of Sigma-Oscillatory Process Model for El Centro 1940 Earthquake Ground Acceleration

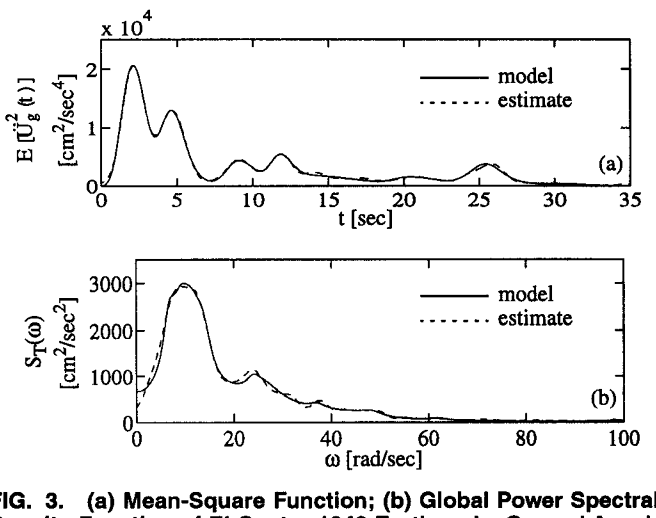

FIG. 3. (a) Mean-Square Function; (b) Global Power Spectral Density Function of El Centro 1940 Earthquake Ground Acceleration

TABLE 2. Ground Motion Parameters for El Centro 1940 Earthquake Record and Statistics from Estimated Ground Motion Model

Parameter (1)	Target (2)	Mean (3)	Standard deviation (4)	COV (5)	Maximum (6)	Minimum (7)
PGA (cm/s^2)	341.695	331.163	52.805	0.159	520.128	232.988
PGV (cm/s)	33.449	40.152	8.952	0.223	66.370	21.695
PGD (cm)	10.867	19.723	6.741	0.342	48.358	8.796
PGV/PGA (s)	0.098	0.123	0.027	0.219	0.185	0.070
PGD/PGA (s^2)	0.032	0.060	0.020	0.329	0.112	0.024
RMSA (cm/s^2)	55.372	55.553	3.475	0.063	62.996	47.510
RMSV (cm/s)	7.645	8.631	1.455	0.169	15.648	6.020
RMSD (cm)	5.309	6.460	2.121	0.328	14.937	2.838
AI (cm/s)	171.882	173.723	21.751	0.125	222.529	126.573
$SI_{0.05}$ (cm)	135.712	130.362	22.940	0.176	190.716	73.360

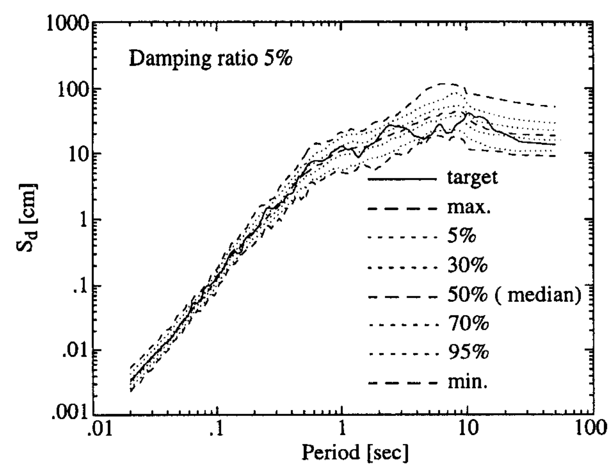

FIG. 4. Probabilistic Linear-Elastic True Relative Displacement Response Spectra for El Centro 1940 Earthquake

Hz and by applying a least-square straight line fitting to both the integrated ground velocity and the displacement records.

Table 2 presents the values of the foregoing ground-motion parameters for the El Centro 1940 target earthquake record and the corresponding second-order statistics generated from the identified earthquake model. The statistical interval defined by ''mean ± one standard deviation'' contains the target parameters, except for PGD and PGD/PGA. Actually, all the target parameters are contained within the interval bounded by the sample maximum and minimum.

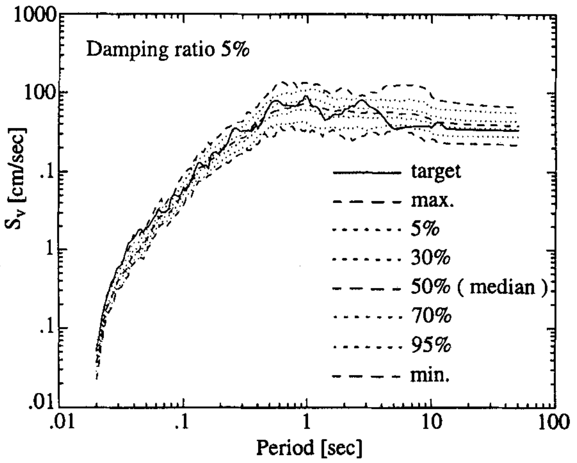

FIG. 5. Probabilistic Linear-Elastic True Relative Velocity Response Spectra for El Centro 1940 Earthquake

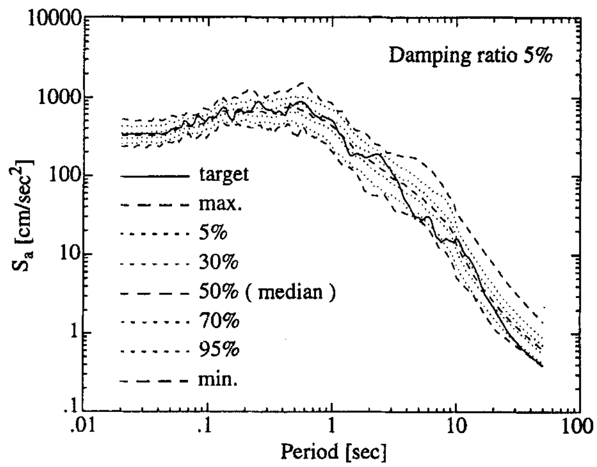

FIG. 6. Probabilistic Linear-Elastic True Absolute Acceleration Response Spectra for El Centro 1940 Earthquake

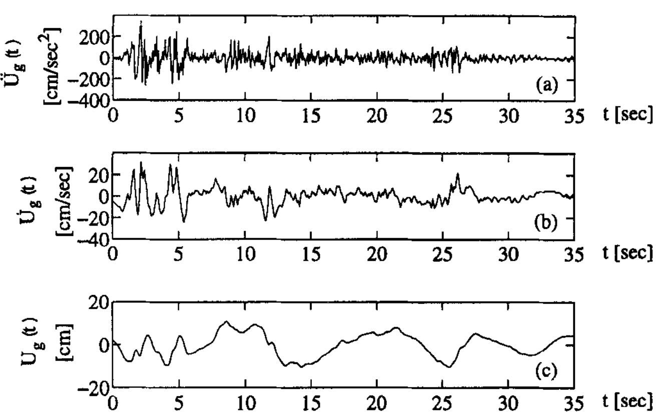

FIG. 7. (a) Actual Ground Acceleration $\ddot{U}_g(t)$; (b) Velocity $\dot{U}_g(t)$; (c) Displacement $U_g(t)$ Time Histories of El Centro 1940 Earthquake Record

The second level of model validation consists of comparing target linear-elastic response spectra with their probabilistic counterparts generated from the identified earthquake model. Figs. 4–6 show the probabilistic linear-elastic response spectra [for the true relative displacement (S_d), true relative velocity (S_v), and true absolute acceleration (S_a) responses] for a probability of exceedence of 95, 70, 50, 30, and 5%. These probabilistic response spectra are obtained from the fractile method of order statistics applied to the sample of 100 response spectra computed from the corresponding artificial ground motions and using the quick numerical algorithm developed by Beck and Dowling (1988). It is observed that in each case the deterministic target spectrum falls within the (5–95%) statistical range of the probabilistic spectrum in the undamped natural

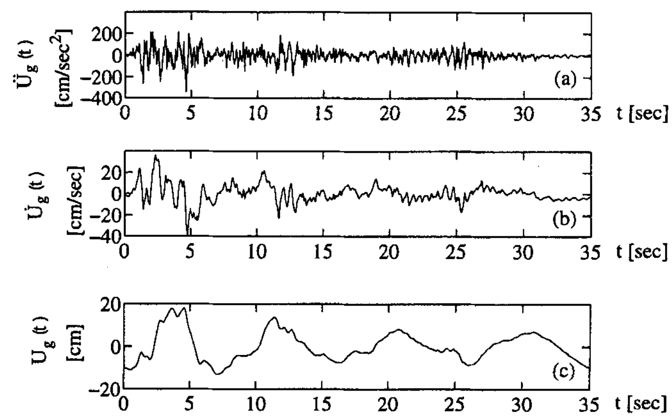

FIG. 8. (a) Artificial Ground Acceleration $\ddot{U}_g(t)$; (b) Velocity $\dot{U}_g(t)$; (c) Displacement $U_g(t)$ Time Histories from El Centro 1940 Earthquake Ground-Motion Model

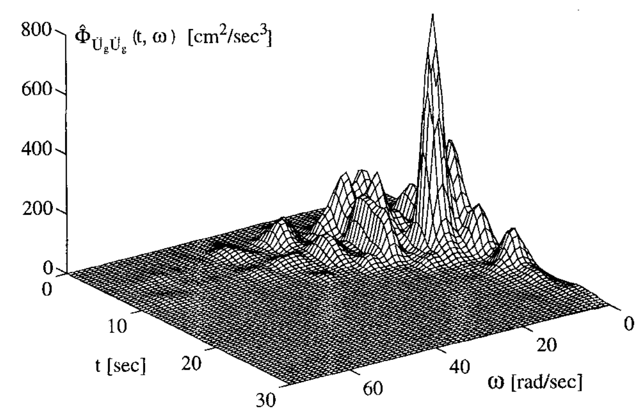

FIG. 9. Estimated Time-Varying Power Spectral Density Function for Orion Boulevard 1971 Earthquake Ground Acceleration

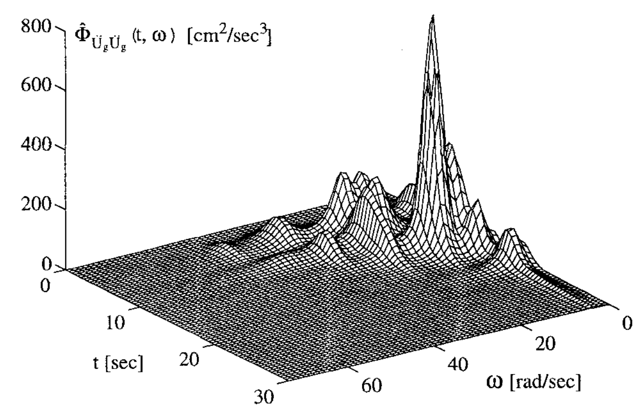

FIG. 10. Analytical Time-Varying Power Spectral Density Function of Sigma-Oscillatory Process Model for Orion Boulevard 1971 Earthquake Ground Acceleration

period interval of practical interest from 0.1 to 2.5 s, which is used to compute the SI_ξ parameter. In fact, the target spectra fall between the sample maximum and minimum for the whole period interval from 0.02 to 50 s.

Figs. 7 and 8 show the ground acceleration, velocity, and displacement time histories from the actual El Centro 1940 record and from a typical artificial ground motion simulated using the identified earthquake model, respectively. It is observed that the artificial ground motions are substantially similar to the actual ones. Based on the results presented, it is concluded that the identified earthquake model captures very well the nonstationary characteristics of the actual El Centro 1940 earthquake record.

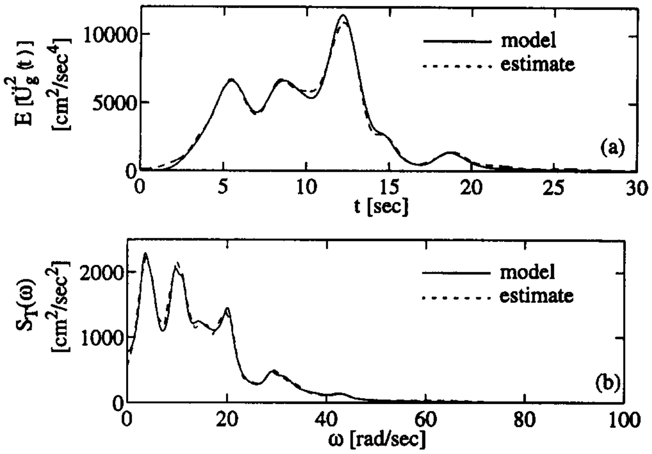

FIG. 11. (a) Mean-Square Function; (b) Global Power Spectral Density Function of Orion Boulevard 1971 Earthquake Ground Acceleration

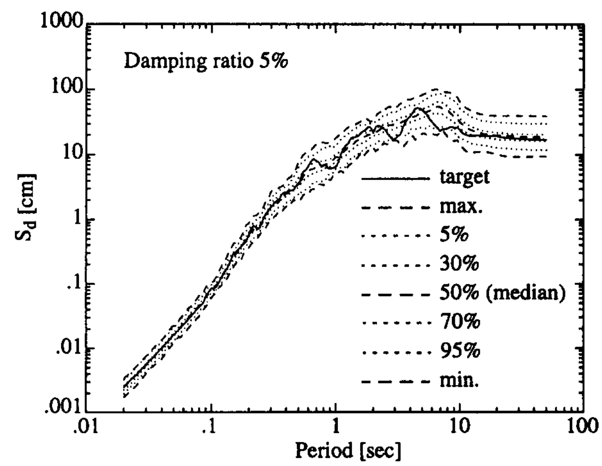

FIG. 12. Probabilistic Linear-Elastic True Relative Displacement Response Spectra for Orion Boulevard 1971 Earthquake

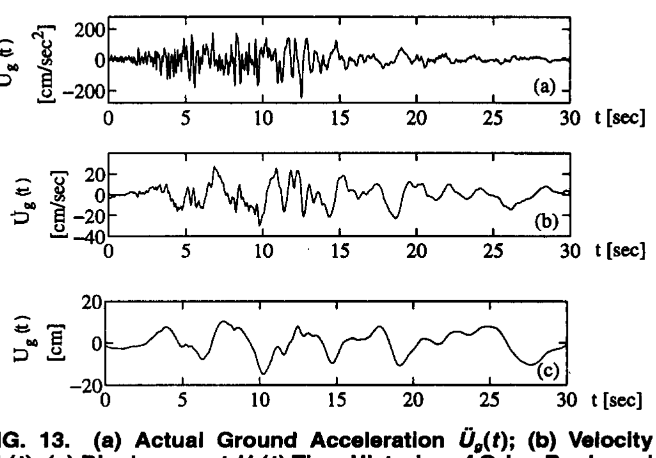

FIG. 13. (a) Actual Ground Acceleration $\ddot{U}_g(t)$; (b) Velocity $\dot{U}_g(t)$; (c) Displacement $U_g(t)$ Time Histories of Orion Boulevard 1971 Earthquake Record

Orion Boulevard 1971, North-South Component

The estimation and validation of the proposed model for the Orion Boulevard 1971 earthquake ground acceleration record are shown in Figs. 9–14 and Tables 3 and 4. Figs. 9 and 10 show that as time elapses, the frequency content of the ground acceleration shifts towards the lower range. This phenomenon is typical of many real earthquake records since the surface waves arriving last have a lower frequency content than the compressive and shear waves arriving earlier.

The results of this second application example demonstrate

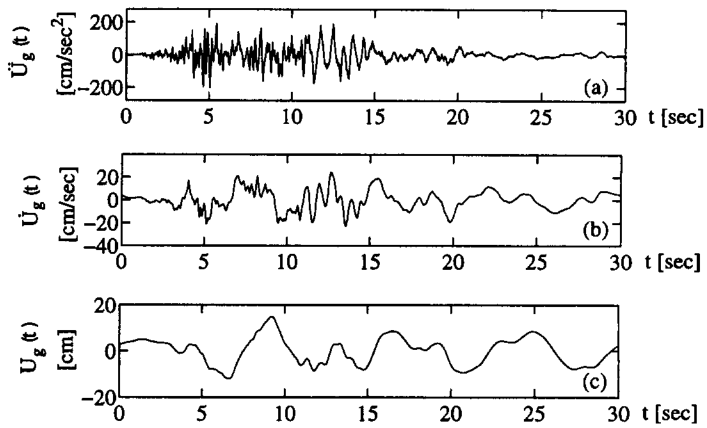

FIG. 14. (a) Artificial Ground Acceleration $\ddot{U}_g(t)$; (b) Velocity $\dot{U}_g(t)$; (c) Displacement $U_g(t)$ Time Histories from Orion Boulevard 1971 Ground-Motion Model

TABLE 3. Estimated Parameters of Ground Acceleration Model for Orion Boulevard 1971 Earthquake Record

k (1)	α_k ( ) (2)	β_k ( ) (3)	γ_k ( ) (4)	ζ_k (s) (5)	ν_k ( ) (6)	η_k (rad/s) (7)	$2\pi B_{\ddot{x}_k}$ (s) (8)
1	0.2358	7	1.3375	0.8672	1.2618	3.1022	13.8356
2	0.1394	8	1.4116	5.5717	1.5856	4.8962	14.2436
3	130.9862	10	4.2028	12.4408	1.9182	4.1720	5.4675
4	3.3724	9	2.5692	15.1802	1.9097	2.7679	8.4034
5	1.0659	2	0.1612	−1.5150	1.2000	3.4588	38.9851
6	71.7647	2	0.8956	9.8679	1.2000	11.0855	7.0158
7	0.0044	11	1.7482	−1.0149	2.0905	14.1349	13.8938
8	0.2012	11	2.4117	3.8821	2.9589	16.4059	10.0712
9	3.7529	11	3.0778	8.7310	1.3580	9.4110	7.8916
10	0.4901	11	2.6406	1.4716	2.4796	19.2596	9.1981
11	12.6339	3	0.7620	6.1032	1.2927	20.2079	12.9527
12	4.1843	5	1.1922	−0.1155	2.7419	31.4748	12.4175
13	5.8917	5	1.3786	5.4048	1.3335	28.7928	10.7389
14	2.1934	5	1.2384	−0.1895	1.8083	43.0850	11.9543
15	20.3968	5	1.7774	5.4490	4.3403	37.5139	8.3293

TABLE 4. Ground Motion Parameters for Orion Boulevard 1971 Earthquake Record and Statistics from Estimated Ground Motion Model

Parameter (1)	Target (2)	Mean (3)	Standard deviation (4)	COV (5)	Maximum (6)	Minimum (7)
PGA (cm/s^2)	249.955	246.556	36.485	0.148	333.623	170.521
PGV (cm/s)	29.998	39.431	8.466	0.215	65.534	24.318
PGD (cm)	14.898	18.859	5.196	0.276	38.136	9.206
PGV/PGA (s)	0.120	0.162	0.035	0.217	0.307	0.087
PGD/PGA (s^2)	0.060	0.078	0.025	0.317	0.179	0.039
RMSA (cm/s^2)	50.927	49.753	3.801	0.076	59.969	42.821
RMSV (cm/s)	9.677	10.160	1.225	0.121	13.487	7.609
RMSD (cm)	5.588	6.766	1.630	0.241	11.226	3.708
AI (cm/s)	124.638	119.663	18.502	0.155	172.854	88.134
$SI_{0.05}$ (cm)	154.457	143.606	22.393	0.156	217.000	97.354

that the nonstationary sigma-oscillatory stochastic model identified faithfully reproduces the intensity and frequency nonstationarity characteristics of the real Orion Boulevard 1971 earthquake record.

CONCLUSIONS

A versatile, fully nonstationary, analytical stochastic earthquake ground-motion model based on the theory of sigma-oscillatory processes was formulated and validated in this paper. First, the time-varying power PSD function of the target real earthquake ground acceleration record was estimated, and denoted as $\hat{\Phi}_{\ddot{U}_g\ddot{U}_g}(t, \omega)$, using the short-time Thomson's multiple-window spectrum estimation method that is consistent, has high resolution, and is not hampered by the usual trade-off between bias (leakage) and variance, which affects the classical nonparametric spectral estimation methods. Then, the stochastic earthquake model corresponding to the target ground motion was built by identifying the "order" of the model (=number of independent component processes) based on the shape of $\hat{\Phi}_{\ddot{U}_g\ddot{U}_g}(t, \omega)$ and estimating the model parameters through an adaptive nonlinear least-squares algorithm. The parameter estimation procedure consists of minimizing the L_2 norm of the error between the analytical time-varying PSD function of the earthquake model, $\Phi_{\ddot{U}_g\ddot{U}_g}(t, \omega)$, and the estimated time-varying PSD function, $\hat{\Phi}_{\ddot{U}_g\ddot{U}_g}(t, \omega)$, the model parameters being subjected to simple inequality constraints.

The proposed earthquake model is physically realizable and can be simulated either by using the nonstationary spectral representation method or by the summation of time-modulated linear combinations of numerically integrated SDOF oscillator responses to independent white-noise processes.

Based on the application examples considered, it is found that the proposed earthquake model is able to capture very well the temporal variation of both the intensity and the frequency content of real earthquake ground motions.

Due to its analytical formulation, the proposed nonstationary earthquake model can be used for analytical linear and nonlinear random vibration studies. This realistic earthquake model is currently used to gain better insight into the effects of the frequency nonstationarity of earthquake ground motions on linear and nonlinear structural response. Validation of the proposed earthquake model has also been demonstrated by comparing target and probabilistic inelastic response spectra. These results will be presented in an upcoming paper.

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APPENDIX I. REFERENCES

Ahmadi, G. (1979). "Generation of artificial time-histories compatible with given response spectra—a review." *SM Archives*, 4(3), 207–239.

Amin, M., and Ang, A. H.-S. (1968). "Nonstationary stochastic model of earthquake motions." *J. Engrg. Mech. Div.*, ASCE, 94(2), 559–583.

Battaglia, F. (1979). "Some extensions in the evolutionary spectral analysis of a stochastic process." *Bolletino della Unione Matematica Italiana*, 16B(5), 1154–1166.

Beck, J. L., and Dowling, M. J. (1988). "Quick algorithms for computing either displacement, velocity or acceleration of an oscillator." *Earthquake Engrg. and Struct. Dyn.*, 16(2), 245–253.

Bogdanoff, J. L., Goldberg, J. E., and Bernard, M. C. (1961). "Response of a simple structure to a random earthquake-type disturbance." *Bull. Seismological Soc. of Am.*, 51(2), 293–310.

Bycroft, G. N. (1960). "White noise representation of earthquakes." *J. Engrg. Mech. Div.*, ASCE, 86(2), 1–16.

Conte, J. P. (1992). "Effects of earthquake frequency nonstationarity on inelastic structural response." *Proc., 10th World Conf. on Earthquake Engrg.*, A. A. Balkema, Rotterdam, The Netherlands.

Conte, J. P., and Peng, B.-F. (1996). "An explicit closed-form solution for linear systems subjected to nonstationary random excitation." *Probabilistic Engrg. Mech.*, 11(1), 37–50.

Conte, J. P., Pister, K. S., and Mahin, S. A. (1992). "Nonstationary ARMA modeling of seismic motions." *J. Soil Dyn. and Earthquake Engrg.*, 11(7), 411–426.

Dennis, J. E., Gay, D. M., and Welsch, R. E. (1981). "An adaptive nonlinear least-squares algorithm." *ACM Trans. on Math. Software*, 7(3), 348–368.

Deodatis, G., Shinozuka, M., and Papageorgiou, A. (1990). "Stochastic wave representation of seismic ground motion. II: Simulation." *J. Engrg. Mech.*, ASCE, 116(11), 2381–2399.

Der Kiureghian, A., and Crempien, J. (1989). "An evolutionary model for earthquake ground motion." *Struct. Safety*, 6(2–4), 235–246.

Drosopoulos, A., and Haykin, S. (1992). "Adaptive radar parameter estimation with Thomson's multiple-window method." *Adaptive radar detection and estimation*, S. Haykin and A. Steinhardt, eds., John Wiley & Sons, Inc., New York, N.Y., 381–461.

Fan, F.-G., and Ahmadi, G. (1990). "Nonstationary Kanai-Tajimi models for El Centro 1940 and Mexico City 1985 Earthquakes." *Probabilistic Engrg. Mech.*, 5(4), 171–181.

Faravelli, L. (1988). "Stochastic modeling of the seismic excitation for structural dynamics purposes." *Probabilistic Engrg. Mech.*, 3(4), 189–195.

Flammer, C. (1957). *Spheroidal wave functions*. Stanford University Press, Stanford, Calif.

Grigoriu, M., Ruiz, S. E., and Rosenblueth, E. (1988). "The Mexico Earthquake of September 19, 1985—nonstationary models of seismic ground acceleration." *Earthquake Spectra*, 4(3), 551–568.

Housner, G. W. (1947). "Characteristics of strong-motion earthquakes." *Bull. Seismological Soc. of Am.*, 37(1), 19–31.

Housner, G. W., and Jennings, P. C. (1964). "Generation of artificial earthquakes." *J. Engrg. Mech. Div.*, ASCE, 90(1), 113–150.

Iyengar, R. N., and Iyengar, K. T. S. R. (1969). "A nonstationary random process model for earthquake accelerograms." *Bull. Seismological Soc. of Am.*, 59(3), 1163–1188.

Kameda, H. (1975). "Evolutionary spectra of seismogram by multifilter." *J. Engrg. Mech. Div.*, ASCE, 101(6), 787–801.

Kanai, K. (1957). "Semi-empirical formula for the seismic characteristics of the ground." *Univ. Tokyo Bull. Earthquake Res. Inst.*, Tokyo, Japan, 35, 309–325.

Kozin, F. (1988). "Autoregressive moving average models of earthquake records." *Probabilistic Engrg. Mech.*, 3(2), 58–63.

Kubo, T., and Penzien, J. (1979). "Simulation of three-dimensional strong ground motions along principal axes, San Fernando Earthquake." *Earthquake Engrg. and Struct. Dyn.*, 7, 279–294.

Levy, R., Kozin, F., and Moorman, R. B. B. (1971). "Random processes for earthquake simulation." *J. Engrg. Mech. Div.*, ASCE, 97(2), 495–517.

Lin, Y. K., and Yong, Y. (1987). "Evolutionary Kanai-Tajimi earthquake models." *J. Engrg. Mech.*, ASCE, 113(8), 1119–1137.

Liu, S. C. (1970). "Synthesis of stochastic representations of ground motions." *The Bell Sys. Tech. J.*, 521–541.

Liu, S. C., and Jhaveri, D. P. (1969). "Spectral simulation and earthquake site properties." *J. Engrg. Mech. Div.*, ASCE, 95(5), 1145–1168.

Papadimitriou, C. (1990). "Stochastic characterization of strong ground motion and applications to structural response." *Rep. No. EERL 90-03*, California Inst. of Technol., Pasadena, Calif.

Priestley, M. B. (1987). *Spectral analysis and time series*. Academic Press, New York, N.Y.

Saragoni, G. R., and Hart, G. C. (1972). "Nonstationary analysis and simulation of earthquake ground motions." *Rep. No. UCLA-ENG-7238*, Earthquake Engrg. and Struct. Lab., Univ. of California, Los Angeles, Calif.

Scherer, R. J., Riera, J. D., and Schueller, G. I. (1982). "Estimation of the time-dependent frequency content of earthquake accelerations." *Nuclear Engrg. and Des.*, 71(3), 301–310.

Shinozuka, M., and Deodatis, G. (1988). "Stochastic process models for earthquake ground motion." *Probabilistic Engrg. Mech.*, 3(3), 114–123.

Shinozuka, M., and Jan, C. M. (1972). "Digital simulation of random processes and its applications." *J. Sound and Vibration*, 25(1), 111–128.

Shinozuka, M., and Sato, Y. (1967). "Simulation of nonstationary random processes." *J. Engrg. Mech. Div.*, ASCE, 93(1), 11–40.

Slepian, D. (1978). "Prolate spheroidal wave functions, Fourier analysis, and uncertainty—V: the discrete case." *The Bell Sys. Tech. J.*, 57(5), 1371–1430.

Spanos, P., Roesset, J., and Donley, M. (1987). "Evolutionary power spectrum estimation of September 19, 1995 Mexico Earthquake accelerograms." *Stochastic Approaches in Earthquake Engrg., Proc., U.S.-Japan Joint Seminar*, Y. K. Lin and R. Minai, eds., Springer-Verlag, New York, N.Y.

Tajimi, H. (1960). "A statistical method of determining the maximum response of a building structure during an earthquake." *Proc., 2nd World Conf. on Earthquake Engrg.*, Vol. II, 781–798.

Thomson, D. J. (1982). "Spectrum estimation and harmonic analysis." *Proc., IEEE*, 70(9), 1055–1096.

Yeh, C.-H., and Wen, Y. K. (1990). "Modeling of nonstationary ground motion and analysis of inelastic structural response." *Struct. Safety*, 8(1–4), 281–298.

Zhang, R., Yong, Y., and Lin, Y. K. (1991). "Earthquake ground motion modeling. II: Stochastic line source." *J. Engrg. Mech.*, ASCE, 117(9), 2133–2148.

APPENDIX II. NOTATION

The following symbols are used in this paper:

$A_k(t)$ = time modulating function of the kth component process;
$A_k(t, \omega)$ = frequency-time modulating function of kth component process;
$D_N(f - \nu)$ = Dirichlet kernel;
$dZ_k(\omega)$ = kth orthogonal increment process;
$E[\ ]$ = ensemble-average or expectation operator;
$H_k(\theta, \omega)$ = Fourier transform of $A_k(t, \omega)$;
$J(\boldsymbol{\theta})$ = error or objective function;
$R_{S_kS_k}(\tau)$ = autocorrelation function of $S_k(t)$;
S_a = linear-elastic true absolute acceleration response spectrum;
S_d = linear-elastic true relative displacement response spectrum;
$S_k(t)$ = kth component stationary Gaussian process;
$S[t_i, n]$ = local-time series centered at time t_i;
$S_k[t_i, \omega_i]$ = kth local-time eigenspectrum centered at time t_i;
$S_T(\omega)$ = global power spectral density function for $\ddot{U}_g(t)$;
S_v = linear-elastic true relative velocity response spectrum;
SI_ξ = Housner spectral intensity with damping ratio ξ;
$\ddot{U}_g(t)$ = earthquake ground acceleration;
$\dot{U}_g(t)$ = earthquake ground velocity;
$U_g(t)$ = earthquake ground displacement;
$v_k(n)$ = kth discrete prolate spheroidal sequence (DPSS);
$w(n)$ = time-moving window;
$w_T(n)$ = time-average window;
$X_k(t)$ = kth oscillatory component process;
$Y(t)$ = sigma-oscillatory process;
$\Gamma(\ )$ = gamma function;
$\delta(\ )$ = Dirac delta function;
$\boldsymbol{\Theta}$ = length-$6p$ parameter vector of the ground-motion model;
ξ = damping ratio of a single-degree-of-freedom (SDOF) oscillator;
$\sigma^2_{t_i}$ = estimated mean square (variance) of $\ddot{U}_g(t_i)$;
$\Phi_{S_kS_k}(\omega)$ = power spectral density function of $S_k(t)$;
$\Phi_{YY}(t, \omega)$ = evolutionary power spectral density function of $Y(t)$;
$\Phi_{X_kX_k}(t, \omega)$ = evolutionary power spectral density function of $X_k(t)$;
$\Phi_{\ddot{U}_g\ddot{U}_g}(t, \omega)$ = evolutionary power spectral density function of $\ddot{U}_g(t)$;
$\hat{\Phi}_{\ddot{U}_g\ddot{U}_g}(t, \omega)$ = estimated evolutionary power spectral density function of $\ddot{U}_g(t)$; and
$\varphi_k(f)$ = kth discrete prolate spheroidal wave function (DPSWF).